M.Sc. PHYSICS
FOURTH SEMESTER
GENERAL THEORY OF RELATIVITY & ASTROPHYSICS
MSP - 402
[USE OMR FOR OBJECTIVE PART]

SET
A

Duration: 3 hrs.

Full Marks: 70

(Objective)

Time: 30 min.

Marks: 20

Choose the correct answer from the following: 1X20=20

1. The interval between two events is called time-like if
   a. $ds^2 > 0$
   b. $ds^2 < 0$
   c. $ds^2 \geq 0$
   d. $ds^2 \leq 0$

2. In a space-time diagram, the angle made by light-like curves with time axis will be
   a. Equal to $\frac{\pi}{4}$
   b. Less than $\frac{\pi}{4}$
   c. Greater than $\frac{\pi}{4}$
   d. All of these

3. The covariant derivative of a second rank tensor becomes a tensor of rank
   a. 1
   b. 2
   c. 3
   d. 4

4. In four-dimensional manifold, the value of the expression $\delta_\sigma^\mu \delta_v^\sigma$ is
   a. 1
   b. 2
   c. 3
   d. 4

5. The conjugate tensor of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^2 \end{pmatrix}$ is

   a. $\frac{1}{r^2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^2 \end{pmatrix}$

   b. $\frac{1}{r^2}\begin{pmatrix} r^2 & 0 & 0 \\ 0 & -r^2 & 0 \\ 0 & 0 & r^2 \end{pmatrix}$

   c. $\frac{1}{r^2}\begin{pmatrix} r^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   d. $\frac{1}{r^2}\begin{pmatrix} r^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & -r^2 \end{pmatrix}$

6. The metric component $g_{\varphi\varphi}$ in the line-element $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$ will be?
   a. $r^2 \sin^2\theta$
   b. r^2
   c. $\sin^2\theta$
   d. 1

7. The number of dependent components in the Einstein tensor $G^{\alpha\beta}$ is
   a. 8
   b. 7
   c. 10
   d. 6

8. The number of independent components in the Riemann curvature tensor $R_{\alpha\beta\gamma\delta}$ is
   a. 15
   b. 19
   c. 17
   d. 20

[1]

9. The assumptions made in the Schwarzschild solution are
   a. The space-time is vacuum
   b. The space-time is asymptotically flat
   c. The space-time is spherically symmetric
   d. All of these

10. If the current time in UTC is 06:40 AM, then the time in IST is
    a. 12:10 AM
    b. 12:10 PM
    c. 5:10 PM
    d. 7:10 AM

11. The South Celestial Pole will be on the horizon for an observer who is at the
    a. north pole
    b. south pole
    c. USTM
    d. equator

12. A star seen at the zenith at USTM must be towards _____ from the Celestial Equator
    a. north
    b. south
    c. east
    d. west

13. The size (diameter) of the Milky Way galaxy is
    a. 1000 LY
    b. 28000 LY
    c. 90000 LY
    d. 100000 LY

14. Which among the following galaxies is most active in star formation?
    a. Irregular
    b. Spiral
    c. Elliptical
    d. Lenticular

15. The equilibrium temperature which both P-P as well as C-N-O cycle possess is found to be
    a. $0.08 \times 10^5 K$
    b. $1.5 \times 10^6 K$
    c. $1.6 \times 10^7 K$
    d. $1.99 \times 10^8 K$

16. If only the gravitational energy runs our Sun, it would power it up only to _____.
    a. 13.8 B Yrs
    b. 50 K Yrs
    c. 30 M Yrs
    d. 10 B Yrs

17. Gravitational redshift is _____ when a photon emitted at a distance $r=R_{sch}$ perpendicularly from the surface of a black hole.
    a. minimum
    b. infinite
    c. zero
    d. none of these

18. Fill in the blank: $p + e \rightarrow$ ____ $+ \upsilon$ (symbols have their usual meaning)?
    a. e^+
    b. γ
    c. H
    d. n

19. The typical density of a star with $1M_0$ converting to a white dwarf leads to
    a. $\rho = 10^6$ gm/cc
    b. $\rho = 10^{11}$ gm/cc
    c. $\rho = 10^{12}$ gm/cc
    d. $\rho = 10^{101}$ gm/cc

20. The _____ of a black hole is the point in space where all the mass of a black hole accumulates.
    a. Point of NO return
    b. Event Horizon
    c. X-ray free zone
    d. Singularity

(**Descriptive**)

Time : 2 hrs. 30 mins.   Marks : 50

[*Answer question no.1 & any four (4) from the rest*]

1. Explain the three main regions of H-R diagram.   5+5=10
   The Luminosity of star Sirius is $25.4L_{\odot}$, and its surface temperature
   $T = 10000K$, find its radius using Stefan's Law. Given $L_{\odot} = 3.85 \times 10^{26}W$.

2. a. Derive an expression the Riemann curvature tensor in terms of   6+2+2
      Christoffel symbols of second kind.   =10

   b. Find the divergence of a scalar function.

   c. Show that covariant derivative of the metric tensor vanish

3. a. Draw the Minkowski space-time diagram showing time-like   4+3+3
      and iight-like curves.   =10

   b. Define time-like and light-like interval.

   c. Using tensor transformation rule, convert the metric $ds^2 = dx^2 + dy^2$ into the cylindrical coordinates.

4. Discuss how a white dwarf form. Explain briefly on electron   5+5=10
   degeneracy pressure.

5. a. If A^{μ} is a tensor, then show that $\partial_{v}A^{\mu} + \Gamma^{\mu}_{\sigma v}A^{\sigma}$ is also a tensor.   4+4+2
                                                                       =10
   b. Express the field equations in terms of Ricci tensor and stress-
      energy tensor.

   c. State the cyclic property of the Riemann curvature tensor.

6. What do you understand by Hydrostatic Equilibrium of a star?   2+4+4
   Establish the relation of Hydrostatic Equilibrium between the pull   =10
   of gravity and outward gas pressure.
   If a star of same size and mass of the Earth converts to a black hole,
   what would be its Schwarzschild radius?

[3]   USTM/COF/R-01

7. a. Find the Christoffel symbols $\Gamma^r_{\mu\nu}$ and $\Gamma^\theta_{\mu\nu}$ using the line-element $$ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2).$$

4+4+2
=10

b. Derive an expression of the effective potential of Schwarzschild vacuum solution.

c. What do you mean by an event horizon?

8. Discuss the steps involve in the following fusion reactions that runs the energy production process in stars.

5+5=10

P-P cycle

C-N-O cycle

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[4]