2023/12

SET A

B.Sc. PHYSICS
FIFTH SEMESTER
ADVANCED MATHEMATICAL PHYSICS
BSP - 504B

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1×20=20***

1. Which of the following is not a vector space?
   a. The set of all real numbers with standard addition and scalar multiplication
   b. The set of all 2x2 matrices with standard matrix addition and scalar multiplication
   c. The set of all polynomials of degree ≤ 3 with polynomial addition and scalar multiplication
   d. The set of all square roots of positive real numbers with standard addition and scalar multiplication

2. If W is a vector subspace of a vector space V, which of the following statements is true?
   a. $\text{Dim}(W) > \text{Dim}(V)$
   b. $W \cap V = \phi$, ϕ is the empty set.
   c. $\vec{0}$ of $W = \vec{0}$ of V
   d. $W \subset V$

3. The set of vectors $\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}$ is considered to be linearly dependent if the equation $\alpha_1\overrightarrow{v_1} + \alpha_2\overrightarrow{v_2} + \alpha_3\overrightarrow{v_3} = 0$ holds for scalars $\alpha_1, \alpha_2, \alpha_3$ such that
   a. $\alpha_1, \alpha_2, \alpha_3 = 0$
   b. α_1 or α_2 or $\alpha_3 \neq 0$
   c. $\alpha_1 = \alpha_2 = \alpha_3$
   d. $\alpha_1 = \alpha_2, \quad \alpha_3 = -\alpha_1$

4. Which of the following sets of vectors forms a linearly independent set in R^3 (four-dimensional Euclidean space)?
   a. {(1, 2, 3), (2, 4, 6), (0, 0, 0)}
   b. {(1, 0, 0), (0, 1, 0), (0, 0, 1)}
   c. {(1, 2, 3), (-3, -6, -9), (0, 0, 0)}
   d. {(1, 1, 1), (2, 2, 2), (3, 3, 3)}

5. Isomorphisms between vector spaces are always:
   a. Surjective.
   b. Bijective
   c. Injective.
   d. None of the above.

6. If a linear transformation is singular, it means that:
   a. It is not defined for all vectors in its domain.
   b. Its range is a proper subset of its codomain.
   c. It is invertible.
   d. It is not invertible.

7. A linear transformation is considered non-singular if:
   a. It has a kernel of dimension greater than zero.
   b. It has a kernel of dimension equal to zero.
   c. Its image is a subset of the codomain.
   d. It is not defined for all vectors in its domain.

8. Which of the following is the correct statement?
a. A scalar is a tensor of rank 0
b. A scalar is a tensor of rank 1
c. A scalar is a tensor of rank 2
d. A scalar is not a tensor

9. The outer product of two mixed tensors $A_{\sigma}^{\mu\nu}$ and B_{β}^{α} followed by contraction will produce a tensor of rank
a. 2
b. 3
c. 1
d. 5

10. In the context of the Kronecker delta, what does the expression $\delta_j^i A_j$?simplify to?
a. A_j
b. A_i
c. $A_i + A_j$
d. A_j^i

11. What is a matrix?
a. A mathematical function
b. A set of real numbers
c. A rectangular array of numbers
d. A type of determinant

12. A square matrix A is idempotent if
a. $A' = A$
b. $A' = -A$
c. $A^2 = A$
d. $A^2 = I$

13. If a square matrix U such that $\overline{U} = U^{-1}$ then U is
a. Orthogonal
b. Unitary
c. Symmetric
d. Hermitian

14. If λ is an eigen value of a non-singular matrix A then the eigen value of A^{-1}
a. $\frac{1}{\lambda}$
b. λ
c. $-\lambda$
d. $-\frac{1}{\lambda}$

15. The sum of the eigen value of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
a. 2
b. 5
c. 7
d. 12

16. Hamilton's equations are a set of first-order differential equations that describe the evolution of
a. Position and velocity
b. Momentum and position
c. Energy and time
d. Angular momentum and time

17. The Lagrangian equation of motion are ____________ order differential equations.

a. First
b. Second
c. Zero
d. Fourth

18. The generalized coordinate has the dimension of velocity, generalize velocity has the dimensions of

a. displacement
b. Velocity
c. acceleration
d. force

19. The generalized coordinates for motion of a particle moving on the surface of a sphere of radius 'a' are ________.

a.
b. $\emptyset$
c. $- \quad \emptyset$
d. $0 \ \emptyset \quad -$

20. The Hamiltonian corresponding to the Lagrangian L= ax^2+by^2-kxy

a. $\frac{2}{2}+\frac{2}{2}+$
b. $\frac{2}{4}+\frac{2}{4}-$
c. $\frac{2}{4}+\frac{2}{4}+$
d. $\frac{2+2}{4}+$

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Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Show that the transformation $P = \frac{1}{2}(p^2 + q^2)$, $Q = tan^{-1}(\frac{q}{p})$ is canonical. **5+5=10**

b. If $A_\sigma^{\mu\nu}$ and $B_\sigma^{\mu\nu}$ are two mixed tensors of rank 3, then prove that their addition and subtraction are also tensors of same rank and same type.

2. a. Show that the set defined by $(x, y, z) \in R^3 : x + 2y = 0, 2x + 3z = 0$ is a subspace of R^3. **3+3+4 =10**

b. If U is defined as, $U = \left\{\begin{bmatrix} x \\ y \end{bmatrix} \in R^2 : \begin{matrix} x+y=0 \\ x-y=1 \end{matrix}\right\}$; check if U is a subspace of R^2.

c. Consider the following sets of vectors in R^3. Hence Determine whether these vectors are linearly dependent or linearly independent.

(i) $\vec{v_1} = (1,2,3)$, $\vec{v_1} = (2,3,5)$, $\vec{v_1} = (3,5,8)$

(ii) $\vec{v_1} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$

3. a. Is the mapping $f : (R^+,\times) \to (R,+)$ an isomorphism? 3+3+4 =10

b. Show that the mapping $f : (Z,+) \to (kZ,+)$ defined by $f(x) = kx$, $\forall x \in Z$ is an isomorphism from Z onto kZ.

c. Determine whether the given mappings exhibit linearity:

(i) $f : V_3(R) \to V_2(R)$, defined by $f(a,b,c) = (a-b, a+c)$.

(ii) $f : V_2(R) \to V_2(R)$, defined by $f(x,y) = (x-1, y+3)$.

4. a. Compute the outer product of the tensors A_j^i and B^m. 2+2+3+ 3=10

b. Define Kronecker delta symbol.

c. Determine the number of independent components for a rank-2 anti-symmetric tensor in three-dimensional space.

d. Prove that the inner product of the tensors A_β^α and B^γ produces a vector.

5. a. Derive the diagonalisation theorem of a matrix. 5+5=10

b. Define the properties of a subspace within a vector space. Prove that the intersection of two subspaces of a vector space is also a subspace.

6. a. Find Lagrange's equation of motion for an electrical circuit comprising an inductance L and capacitance C. 5+5=10

The capacitor is charged to q coulombs and current flowing in the circuit is i amperes.

b. For a system with the Lagrangian $L = \frac{1}{2}(q_1^2 + q_1q_2 + q_2^2) - V(q)$, show that the Hamiltonian is $H = \frac{2}{3}(p_1^2 - p_1p_2 + p_2^2) + V(q)$

7. a. Obtain the equation of motion of two masses, connected by an inextensible string passing over a small smooth pulley. 5+5=10

b. State and prove the D'Alembert's principle.

8.

a. If λ be an Eigen value of a matrix A, then prove that $\lambda+k$ is an Eigen value of $A+kI$. 2+8=10

b. Solve $\frac{d^2x}{dt^2}+4\frac{dx}{dt}-12x=0.$

$x(0)=0, x'(0)=8$ by matrix method.

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