REV-01
BSM/05/10

2023/12

B.SC. MATHEMATICS
FIRST SEMESTER
ALGEBRA
BSM – 102

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(PART-A: Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×20=20***

1. The polar form of i is
   a. $e^{\frac{i\pi}{2}}$
   b. $e^{\frac{-i\pi}{2}}$
   c. $e^{i2\pi}$
   d. 0

2. The real and imaginary part of $-i$ are respectively
   a. 0 and 1
   b. 1 and 0
   c. 0 and -1
   d. -1 and 0

3. For $z = \sin\theta - i\cos\theta$, the value of mod z is
   a. 0
   b. 1
   c. $\sin\theta$
   d. $\cos\theta$

4. If $z = \frac{1-i}{1+i}$ then the conjugate of z is
   a. $\frac{1+i}{1-i}$
   b. i
   c. $-i$
   d. 1

5. If $\begin{bmatrix} x-2 & 3 & 2z \\ 6y & x & 2y \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 18z & y+2 & 6z \end{bmatrix}$, then the value of (x, y, z) is equal to
   a. $(11,3,9)$
   b. $(11,9,3)$
   c. $(3,9,11)$
   d. None of these

6. The remainder when $x^5 + 2x^4 + x^3 + 5x^2 + 2x + 11$ is divided by $x + 1$ is
   a. 10
   b. 22
   c. -10
   d. -22

7. If α is a zero of order r of the polynomial $f(x)$ then
   a. $(x-\alpha)^r$ is a factor of $f(x)$.
   b. $(x-\alpha)^{r+1}$ is a factor of $f(x)$.
   c. Both $(x-\alpha)^r$ and $(x-\alpha)^{r+1}$ are factors of $f(x)$.
   d. $(x-\alpha)^r$ is a factor of $f(x)$ but $(x-\alpha)^{r+1}$ is not a factor of $f(x)$

8. If $f(x) = x^4 + px^2 + qx + r$ has a factor of the form $(x-\alpha)^3$ then (Here $f^n(x)$ denotes the nth derivative of $f(x)$.)
   a. $f^3(\alpha) = 0$
   b. $f^3(\alpha) \neq 0$
   c. $f^2(\alpha) \neq 0$
   d. None of these

9. If α is a multiple root of the polynomial equation $f(x) = 0$ of order r then

a. α is a multiple root of the polynomial equation $f'(x) = 0$ of order r

b. α is a multiple root of the polynomial equation $f'(x) = 0$ of order $r + 1$

c. α is a multiple root of the polynomial equation $f'(x) = 0$ of order $r - 1$

d. α is not a multiple root of the polynomial equation $f'(x) = 0$

10. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then the transpose of A^2 is

a. $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

c. $2A$

d. None of these

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(PART-B : Descriptive)

Time : 1 hr. 15 mins. Marks : 25

[Answer question no.1 & any two (2) from the rest]

1. If $x^3 + 3px + q$ has a factor of the form $(x - \alpha)^2$ show that $q^2 + 4p^3 = 0$. 5

2. (a) Find the remainder when $x^5 - 3x^4 + 4x^2 + x + 4$ is divided by $(x + 1)(x - 2)$. 5+5=10

(b) Let $f(x) = x^4 - x^3 + 2x^2 + 6x - 2$. Use the method of synthetic division to find $f(x + 2)$.

3. (a) If z_1, z_2 are two complex numbers, then prove that 4+3+3 =10

$$|z_1 + z_2| \le |z_1| + |z_2|$$

(b) Express z in polar form, where

(i) $z = 1 - i$

(ii) $z = -1 - i$

4. (a) If n be an integer, prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$ 5+5=10

(b) Fine the real and imaginary part of $z = \frac{2+3i}{2+i}$. Also, find the conjugate of z.

5. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$ **5+5=10**

(b) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation

$$A^3 - 4A^2 + A = 0.$$

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