REV-01
BSM/02/05

2023/12

SET A

B.Sc. MATHEMATICS
THIRD SEMESTER
CLASSICAL ALGEBRA & TRIGONOMETRY
BSM - 731[REPEAT]

[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. Full Marks : 70

(Objective)

Time : 30 min. Marks : 20

Choose the correct answer from the following: ***1X20=20***

1. If α, β, γ are roots of the equation $x^3 + qx + r = 0, then\ \sum \alpha\beta = ?$
   a. q
   b. $-r$
   c. r
   d. none

2. The condition $AM = GM$ holds when the quantities are
   a. equal
   b. unequal
   c. Hold for any numbers
   d. none

3. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then what is the value of $\sum \alpha^2\beta$
   a. $-3r$
   b. $3r$
   c. $2r$
   d. none

4. In which case a system of equations has no solution?
   a. (adj A).B$\neq$ 0
   b. (adj A).B = 0
   c. (adj A). B $<$ 0
   d. none

5. The value of i^i is
   a. $e^{-(4n+1)\frac{\pi}{2}}$
   b. $e^{-(4n+1)\frac{\pi}{3}}$
   c. $e^{-(4n+1)\frac{\pi}{8}}$
   d. none

6. The value of $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = ?$, where ω is the cube root of unity
   a. 1
   b. 2
   c. -1
   d. 0

7. If the sum of two roots of the equation $x^3 + a_1x^2 + a_2x + a_3 = 0\ is\ zero, then$
   a. $a_1a_2 = -a_3$
   b. $a_1a_2 = a_3$
   c. $a_2 = a_3$
   d. none

8. If a, b, c are positive numbers then, which of the following is true?
   a. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 3$
   b. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 9$
   c. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$
   d. none

9. Find the determinant of M= $\begin{bmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{bmatrix}$

a. 1 b. θ

c. $bc + ca + ab$ d. none

10. Find x, y, z *and* t *which satisfy the matrix equation*

$$\begin{bmatrix} x-y & 2x+z \\ 2x-y & 2z+t \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

a. $x = 1, y = 2, z = 3, t = -1$ b. $x = 1, y = -2, z = 3, t = -1$

c. $x = 1, y = 2, z = 3, t = 7$ d. none

11. If z is a non-zero complex number, then $z\bar{z}$ *is*

a. Purely real b. Purely imaginary

c. zero d. none

12. Which of the following is not a symmetric function?

a. $x^2 + y^2 + z^2$ b. $x + y$

c. $x - y$ d. xy

13. If α, β are roots of $x^2 - 2x + 4 = 0$, then what is the value of α^n?

a. $2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$ b. $2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right)$

c. $2^{n+1}\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$ d. none

14. Which of the following will not hold for any positive real numbers belongs to R, if $a > b$,then

a. $a + c > b + c$ b. $a - c > b - c$

c. $ac < bc$ d. none

15. Which inequality is known as Cauchy- Schwartz's inequality?

a. $(\sum_{i=1}^{n} a_i b_i) \geq (\sum_{i=1}^{n} a_i)\ (\sum_{i=1}^{n} b_i)$ b. $(\sum_{i=1}^{n} a_i b_i)^2 \leq (\sum_{i=1}^{n} a_i^2)\ (\sum_{i=1}^{n} b_i^2)$

c. $(\sum_{i=1}^{n} a_i^2 b_i^2) \leq (\sum_{i=1}^{n} {a_i}^2)(\sum_{i=1}^{n} b_i^2)$ d. none

16. Which is the correct expression for Gregory series?

a. $\theta = \tan\theta + \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} \ldots.$ b. $\theta = \tan\theta - \frac{\tan^3\theta}{3} - \frac{\tan^5\theta}{5} \ldots.$

c. $\theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} \ldots.$ d. none

17. Evaluate the value of $\frac{(\cos\theta - i\sin\theta)^{10}}{(\cos\theta + i\sin\theta)^{12}}$

a. $\cos 22\theta - i\sin 22\theta$ b. $\cos 22\theta + i\sin 22\theta$

c. $\cos 12\theta - i\sin 12\theta$ d. none

18. The value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is

a. $(a-b)(b-c)(c-a)$

b. $-(a-b)(b-c)(c-a)$

c. $(a-b)(b-c)$

d. none

19. If $x = 5 + 2i, y = 5 - 2i, then\ x^2 + y^2 + xy = ?$

a. -71

b. 71

c. 17

d. none

20. Find the cofactor of 6 of the determinant $\begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$

a. -4

b. 5

c. 4

d. none

-- --- --

Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Define polynomial and fundamental theorem of Arithmetic. Find the conditions under which the roots of the equation $x^3 + px^2 + qx + r = 0$ are in (i) AP (ii) GP **2+8=10**

2. a. State Cauchy Schwartz Inequality. If $a^2 + b^2 + c^2 = 1$ then show that $-\frac{1}{2} \leq ab + bc + ca \leq 1$. **1+4+5 =10**

   b. If a, b, c are positive and distinct show that $\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b} > 6$.

3. a. State and prove De Moivre's Theorem. **8+2=10**

   b. Give the exponential expansion of Sine and Cosine

4. a. Define upper triangular, lower triangular and diagonal matrix with example. 6+4=10

b. Find the rank of the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

5. a. State and deduce Gregory's series . 4+3+3 =10

b. (i) Prove that $sin\left(i\ log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2+b^2}$

(ii) Prove that $\log(1 + i\ tan\theta) = \log sec\theta + i\theta$

6. a. Solve the system of equation by matrix method 6+4=10

$$x + 2y + 3z = 4,$$
$$4x + 4y + 9z = 6$$
$$x + y + z = 3.$$

b. If A= $\begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ find AB' and BA'.

7. a. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ given that the roots are in geometric progression. 5+5=10

b. If $\alpha, \beta$and γ are the roots of the equation $x^3 + px + q = 0$ then find the vales of $\sum \alpha^2$ and $\sum \frac{1}{\alpha+\beta}$.

8. a. If $\tan(x + iy) = u + iv$ then prove that $u^2 + v^2 + 2ucot2x = 1$. 4+3+3 =10

b. (i) Prove that $(1 + cos\theta + isin\theta)^n = 2^n \cos^n\frac{\theta}{2}(cos\frac{n\theta}{2} + isin\frac{n\theta}{2})$.

(ii) Determine the equation in complex form of a circle with centre
(-3, 4) and radius 2.

= = *** = =