REV-01
MSM/25/30

2023/12

SET A

M.Sc. MATHEMATICS
FIRST SEMESTER
LINEAR ALGEBRA
MSM – 102

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. Full Marks: 70

(Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: ***1×20=20***

1. Let V be a vector space of all real numbers R over R. Which of the following is vector subspace of V?
   a. $C(R)$
   b. $R(R)$
   c. $Q(R)$
   d. $Z(R)$

2. Which one of the following polynomials lies in the linear span of $S=\{1,1+x+x^2\}$?
   a. $5x^2+5x+1$
   b. $100x^2+10x+10$
   c. $\pi(x^2+1)$
   d. $7x^2+\pi x+3$

3. The singleton set $\{\alpha\}$ is linearly dependent iff
   a. $\alpha=0$
   b. $\alpha\neq 0$
   c. α is scalar
   d. None

4. If W is the proper subspace of a finite dimensional vector space V , then
   a. $\dim V<\dim W$
   b. $\dim W<\dim V$
   c. $\dim V=\dim W$
   d. None

5. Dimension of vector subspace $W=\{(a,b,c):a=b=c\}$ *over vector space* $R^3(R)$ equals
   a. 0
   b. 1
   c. 2
   d. 3

6. If $U=\{(x,y,z):x=y=z\}, V=\{(x,y,z):x=0\}$. Then what is $U+V$
   a. xy- plane
   b. yz- plane
   c. zx- plane
   d. R^3

7. If I is an identity transformation on finite vector space V. Then nullity of I is
   a. dimV
   b. Zero
   c. 1
   d. None

8. Let V be a finite dimensional space. T is a zero transformation on V. Then range of T is

a. V
b. $\{0\}$
c. ϕ
d. None

9. Let V be a vector space. T is a linear transformation of V into V such that $T(\alpha)=\alpha, \alpha \in V$ then T is

a. Identity transformation
b. Zero transformation
c. Invertible transformation
d. Orthogonal transformation

10. Which of the following is a 2-dimensional subspace of R^3 $over$ R

a. $\{(0,x,0): x \in R\}$
b. $\{(0,x,0): x \in R\} \cup \{(0,0,y): y \in R\}$
c. $\{(x,y,0): x,y \in R \; and \; x+y=0\}$
d. $\{(0,x,z): x,z \in R\}$

11. A set containing linearly dependent set is

a. Linearly independent
b. Linearly dependent
c. Null set
d. None

12. If α is a characteristic root of a non-singular matrix A, then characteristic root of adj(A) is

a. $\alpha|A|$
b. α
c. $\dfrac{|A|}{\alpha}$
d. $\dfrac{|adjA|}{\alpha}$

13. Let P be a 4×4 matrix whose determinant is 10. The determinant of the matrix -3P is

a. -810
b. -30
c. 30
d. 810

14. Let P and Q be two $n\times n$ non-zero matrices such that P+Q=0. Which one of the following statements is never true?

a. P is non-singular
b. $P=Q^t$
c. $P=Q^{-1}$
d. $Rank(P)\neq Rank(Q)$

15. The number of values of λ for which the system of equations

$\lambda x+(\lambda+3)y=10z$

$(\lambda-1)x+(\lambda-2)y=5z$ has infinitely many solutions, is

$2x+(\lambda+4)y=\lambda z$

a. 1
b. 2
c. 3
d. infinite

16. An orthogonal set of non-zero vectors is

a. Linearly independent
b. Linearly dependent
c. Constant
d. None

17. The orthogonal complement of inner product space V is

a. Zero subspace
b. V itself
c. Any subspace
d. None

18. The Cauchy-Schwarz inequality states

a. $|<\alpha\beta>| \geq \|\alpha\|\|\beta\|$
b. $|<\alpha\beta>| \leq \|\alpha\|\|\beta\|$
c. $\|(\alpha+\beta)\| \geq \|\alpha\|\|\beta\|$
d. None

19. Let S be an orthonormal set then for $\alpha \in S$

a. $\|\alpha\| = 0$
b. $\|\alpha\| > 0$
c. $\|\alpha\| = 1$
d. $\|\alpha\| < 1$

20. If V be a vector space then V is

a. Field
b. Abelian group
c. Null set
d. None

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(Descriptive)

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Find the eigen values and eigen vectors of the matrix 10

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

2. State Cayley-Hamilton theorem. Find the characteristic equation of the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and verify Cayley-Hamilton theorem. 1+7+2 =10

Hence evaluate the matrix equation

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$$

3. Prove that a linearly independent subset of a finitely generated vector space is either a basis or can be extended to form a basis. 10

4. If T is an operator on R^3 whose basis is $B=\{(1,0,0),(0,1,0),(0,0,1)\}$ such that 10

$[T:B]=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & -1\\ -1 & -1 & 0\end{bmatrix}$. Find the matrix of T with respect to basis

$$B_1=\{(0,1,-1),(1,-1,1),(-1,1,0)\}$$

5. Show that the matrix $A=\begin{bmatrix}1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ is diagonizable. 10

6. Verify whether the transformation is linear from R^2 into R^3. Find the range, rank, null space and nullity for the transformation $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ **2+2+4+2=10**

7. State and prove Cauchy-Schwarz's inequality. **10**

Or

If $C(x) = (x-3)^3 (x-2)^4$ and $m(x) = (x-3)^2 (x-2)^2$, then find all possible Jordan canonical forms of the given characteristic and minimal polynomials.

8. a. Show that the vectors $(0,1,-2), (1,-1,1), (1,2,1)$ form a linearly independent set. **5+5=10**

   b. Find the dimension of the solution space W of the system of linear equations

$x + 2y - 4z + 3r - s = 0$

$x + 2y - 2z + 2r + s = 0$

$2x + 4y - 2z + 3r + 4s = 0$

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