REV-01
MSM/24/29

2023/12

**SET A**

# M.Sc. MATHEMATICS
## THIRD SEMESTER
## NUMBER THEORY
## MSM – 302

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. Full Marks: 35

**(Objective)**

Time: 15 mins. Marks: 10

***Choose the correct answer from the following:*** ***1X10=10***

1. The remainder obtained when $16^{2016}$ is divided by 9 equals
   a. 1
   b. 3
   c. 5
   d. 7

2. Which of the following congruence has no solution
   a. $37x \equiv 1 \pmod{12}$
   b. $5x \equiv 2 \pmod{26}$
   c. $6x \equiv 15 \pmod{21}$
   d. $39x \equiv 1 \pmod{13}$

3. Which of the following primes satisfy the congruence $a^{24} - 6a - 2 \equiv 0 \pmod{13}$?
   a. 41
   b. 67
   c. 83
   d. None of these

4. Consider the congruence $x^n \equiv 2 \bmod 13$. This congruence has a solution if
   a. $n = 4$
   b. $n = 5$
   c. $n = 6$
   d. None of these

5. Which of the following equation have $[\overline{1;2}]$ as a continued fraction representation
   a. $2x^2 - 2x + 1 = 0$
   b. $2x^2 - 2x - 1 = 0$
   c. $2x^2 + 2x - 1 = 0$
   d. None of these

6. Which of the following is/are convergent of $[2; 3, 2, 5, 2, 4, 2]$
   a. $\frac{78}{38}$
   b. $\frac{191}{83}$
   c. $\frac{847}{370}$
   d. $\frac{1885}{823}$

7. If $p$ is a factor of $2^{\frac{p-1}{2}} + 1$ then
   a. $p \equiv 7 \pmod{8}$
   b. $p \equiv 5 \pmod{8}$
   c. $p \equiv 1 \pmod{8}$
   d. None of these

8. The value of $[0; 1, 1, 1, 1, \cdots 1]$ is
   a. $\frac{F_{n+1}}{F_n}$ ($F_n$ denotes the $n$th Fibonacci number)
   b. $\frac{F_n}{F_{n+1}}$ ($F_n$ denotes the $n$th Fibonacci number)
   c. $\frac{1-\sqrt{5}}{2}$
   d. $\frac{1+\sqrt{5}}{2}$

9. The value of $\gcd(F_{14}, F_{39})$ is

a. 1

b. $F_{11}$

c. 21

d. 89

10. Which of the following is/are true?

a. 6 is the integer root of $x^2 + x + 1 \equiv 0 \pmod{7}$

b. 6 is not the integer root of $x^2 + x + 2 \equiv 0 \pmod{7}$

c. $67 \equiv 6 \pmod{7}$

d. $6^7 \not\equiv 6 \pmod{7}$

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## (Descriptive)

Time : 1 hr. 15 mins. Marks : 25

*[Answer question no.1 & any two (2) from the rest]*

1. Find the value of $[3; \overline{1,1,1,1,6}]$. **5**

2. **6+4=10**

   a. Prove that - $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ where $F_n$ denotes the $n$th Fibonacci numbers.

   b. Prove that-

$$\left(\frac{-2}{p}\right) = \begin{cases} 1 & \text{if } p \equiv 1 \pmod 8 \text{ or } p \equiv \pmod 8 \\ -1 & \text{if } p \equiv 1 \pmod 8 \text{ or } p \equiv \pmod 8 \end{cases}$$

3. **2+6+2 =10**

   a. Prove that - $\phi(2^n - 1)$ is a multiple of $n$ for any $n > 1$.

   b. Solve the congruence $x^3 \equiv 5 \pmod{13}$.

   c. Solve the congruence $7^x \equiv 7 \pmod{13}$.

4. **4+3+3 =10**

   a. Prove that- 3 is a primitive root of any prime of the form $2^{2^n} + 1, n > 1$.

   b. Evaluate the Legendre symbol $\left(\frac{3658}{12703}\right)$.

   c. For an odd prime $p$, prove that the congruence $2x^2 + 1 \equiv 0 \pmod p$ has a solution if and only if $p \equiv 1$ or $3 \pmod 8$.

5. **8+2=10**

   a. Find the smallest positive integer solution of $x^2 - 14y^2 = 1$.

   b. Prove that: $F_1 + F_2 + F_3 + \cdots + F_n = F_{n+2} - 1$, here $F_n$ denotes the $n$th Fibonacci number.

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