SET A

M.Sc. PHYSICS
FOURTH SEMESTER
THEORY OF RELATIVITY-II
MSP-403E

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. Two photons approach each other. Their relative velocity will be
   a. c b. 0
   c. c/2 d. 2c

2. The velocity of a rocket ship is v=0.1c. The percentage of decrease in length will be?
   a. 10% b. 98%
   c. 99% d. 49%

3. Which one of the following expression is correct for the surface charge density?
   a. $\sigma' = \sigma\sqrt{1-\beta^2}$ b. $\sigma' = \sqrt{1-\beta^2}$
   c. $\sigma' = \sigma/\sqrt{1-\beta^2}$ d. $\sigma' = \sigma$

4. If $A^\mu = (0,\ r^2)$ and $g_{\mu\upsilon} = \text{diag}(-1,\ r^{-2})$ then A_μ will be
   a. (0, 1) b. (1, 0)
   c. (-1, 1) d. (1, 1)

5. The determinant of the metric tensor $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^2 \end{pmatrix}$ is
   a. -1 b. r^2
   c. $-r^2$ d. 1

6. The product of $g_{\varphi\varphi}\ g^{\varphi\varphi}$ in the line-element $ds^2 = dr^2 + \alpha^2\ r^2 d\varphi^2$ will be?
   a. $\alpha^2\ r^2$ b. $\frac{1}{\alpha^2 r^2}$
   c. $\frac{\alpha^2}{r^2}$ d. 1

7. The number of independent components in the Einstein tensor $G^{\alpha\beta}$ will be?
   a. 8 b. 7
   c. 10 d. 6

8. The Riemann curvature tensor $R^\alpha{}_{\beta\gamma\delta}$ is a tensor of rank
   a. 2 b. 4
   c. 3 d. 1

9. The rank of the mixed tensor $\delta^\tau_\alpha \delta^\mu_\sigma A^\sigma_{\rho\tau}$ is
   a. 2 b. 3
   c. 4 d. 5

10. The Ricci tensor $R^{\alpha\beta}$ is a tensor of rank
    a. 3 b. 4

c. 1 d. 2

11. The Hubble parameter H is defined by
    a. $\frac{a(t)}{\dot{a}(t)}$ b. $\frac{\dot{a}(t)}{a(t)}$
    c. $\frac{1}{\dot{a}(t)}$ d. $\dot{a}(t)$

12. In FRW metric, the value of k for which the geometry represents a closed space will be?
    a. k=0 b. k=1
    c. K=-1 d. k=2

13. For a radiation fluid, the pressure p is related with the energy density ρ by
    a. $p = \frac{1}{3}\rho$ b. p= ρ
    c. $\rho = \frac{1}{3}p$ d. $p = -\rho$

14. A vector X^μ is said to be null if
    a. $X^\mu X_\mu < 0$ b. $X^\mu X_\mu > 0$
    c. $X^\mu X_\mu = 0$ d. None of these

15. If the Ricci tensor $R^{\alpha\beta}$ is a symmetric and 2nd rank tensor then the Einstein tensor $G^{\alpha\beta}$ is
    a. a symmetric and 2nd rank b. anti-symmetric and 2nd rank
    c. symmetric and 1st rank d. anti-symmetric and 1st rank

16. Which one of the following option is correct?
    a. $A^\mu_{;\mu} = \frac{1}{g}\frac{\partial}{\partial x^\mu}(\sqrt{g}A^\mu)$ b. $A^\mu_{;\mu} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\mu}(\sqrt{g}A^\mu)$
    c. $A^\mu_{;\mu} = \frac{1}{\sqrt{g}}\frac{\partial A^\mu}{\partial x^\mu}$ d. $A^\mu_{;\mu} = \frac{1}{g}\frac{\partial}{\partial x^\mu}(gA^\mu)$

17. For an Einstein in de-Sitter space-time, the cosmological constant Λ is
    a. $\Lambda > 0$ b. $\Lambda = 0$
    c. $\Lambda < 0$ d. None of these

18. If the norm of a vector X_μ is time-like and the Lie derivative of the metric tensor vanish, $L_X g_{\mu\nu} = 0$, then the vector is said to be
    a. Light-like Killing vector b. Space-like Killing vector
    c. Time-like Killing vector d. None of these

19. The electromagnetic field tensor $F_{\mu\nu}$ is defined by
    a. $\partial_\mu A_\nu$ b. $\partial_\nu A_\mu - \partial_\mu A_\nu$
    c. $\partial_\mu A_\nu - \partial_\nu A_\mu$ d. $\partial_\nu A_\mu$

20. For vacuum field equations with zero cosmological constant, which one of the following option is correct?
    a. $G_{\mu\nu} = 0$ but $R_{\mu\nu} \neq 0$ b. $R_{\mu\nu} = 0$ and $R = 0$
    c. $R_{\mu\nu} \neq 0$ and $R = 0$ d. $G_{\mu\nu} = 0$ and $R_{\mu\nu} = 0$

Descriptive

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. What do you meant by a metric tensor? **2+8=10**

   b. By considering the variation of $\int ds$, derive the geodesic equation

   $$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{ds}\frac{dx^\sigma}{ds} = 0.$$

2. a. Derive the relation between the Ricci tensor $R_{\alpha\beta}$ and the stress-energy tensor $T_{\alpha\beta}$ **5+2+3 =10**

   b. Find the condition for vacuum Field equations.

   c. What do you mean by perfect fluid? Write its energy momentum tensor.

3. a. Define Christoffels 3-index symbols. **2+4+4 =10**

   b. Find the divergence of a contravariant vector A^μ.

   c. If A_μ is a tensor, then show that $\partial_\nu A_\mu - \Gamma^\sigma_{\mu\nu}A_\sigma$ is also a tensor. What is the nature of this tensor?

4. a. What do you mean by Killing vector? **2+4+4 =10**

   b. Show that Lie derivative of a metric tensor w. r. t. Killing vector vanishes.

   c. Derive the Killing equation of a metric tensor.

5. a. Show that $G^\mu_{\nu\,;\,\mu} = 0$, where G^μ_ν is the Einstein tensor. **6+2+2 =10**

   b. Write down the relation between the Einstein tensor and the Ricci tensor.

   c. What is the cyclic property of the Riemann curvature tensor?

6. a. Show that in a weak field approximation the geodesic equation reduces to the Newton equation. **5+3+2 =10**

   b. Explain the red shift phenomena of the Schwarzschild vacuum solution.

   c. What is the experimental value of bending of light in Schwarzschild vacuum solution?

7. **a.** For a vacuum field equations, find A(r) and B(r) using the following spherically symmetric metric **6+2+2 =10**

$$ds^2 = -A\,(r)dt^2 + B\,(r)dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2)$$

b. Under what condition the above line-element becomes flat space?

c. For a Schwarzschild solution writes the function A(r) and B(r).

8. **a.** What is Cosmology? **2+6+2 =10**

b. Derive the Friedmann equation $\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{\rho}{3}$, where symbols have their usual meanings.

c. Define Hubble parameter.

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