M.Sc. MATHEMATICS
FOURTH SEMESTER
ADVANCED PARTIAL DIFFERENTIAL EQUATION
MSM – 402 [SPECIAL REPEAT]
[USE OMR FOR OBJECTIVE PART]

SET A

Duration : 3 hrs.

Full Marks : 70

(Objective)

Time : 30 min.

Marks : 20

Choose the correct answer from the following: 1X20=20

1. An equation of the form $Ru_{xx}+Su_{xy}+Tu_{yy}=f$, is parabolic if S^2-4RT
   - a. >0
   - b. <0
   - c. Equals 0
   - d. None of these

2. A general equation of the form $4u_{xx}-u_{yy}=0$ is of the form $u(x,y)=?$
   - a. $f(x)+g(y)$
   - b. $f(x+2y)+g(y-2y)$
   - c. $f(x+4y)+g(x-4y)$
   - d. None of these

3. Euler's equation reduces to $F_{y'}=0$, if
   - a. $F = F(x,y)$
   - b. $F = F(x,y')$
   - c. $F = F(y,y')$
   - d. None of these

4. Shortest distance between two fixed points in the Eiclidean plane is
   - a. Cardiod
   - b. Jordan curves
   - c. Straight lines
   - d. None of these

5. The problem of finding a closed curve of given length which encloses maximum area is called
   - a. Extremal problem
   - b. Maximal problem
   - c. Isoperimetric problem
   - d. None of these

6. For any function $u(x,y)$, the equation $uxx+uyy=0$ is called
   - a. Neumann eqution
   - b. Heat equation
   - c. Wave equation
   - d. Laplace equation

7. $Y_{xx} - Y_{tt} = 0$ is
   - a. Two-dimensional heat equation
   - b. Two-dimensional wave equation
   - c. One-dimensional heat equation
   - d. One-dimensional wave equation

8. The equation $r2R'' + rR' + m2r2R = 0$ is Bessel's equation of
   - a. Zeroth order
   - b. First order
   - c. Order not defined
   - d. Second order

9. The equation $r^2R'' + rR' - m^2R = 0$ is
   - a. Homogeneous linear
   - b. Homogeneous non-linear
   - c. Non-homogeneous linear
   - d. Non-homogeneous non-linear

USTM/COE/R-

10. Consider the wave equation Utt=4Uxx , where 0<x<1800, t>0 with U(0,t)=U(1800,t)=0, U(x,0)=sinx and Ut=0 at t=0. Then U(900,900) is
    a. 2
    b. 1
    c. 0
    d. -1

11. If a real function f satisfies Dirichlet's conditions over an interval (-m, m), then f is periodic with period
    a. m
    b. 2m
    c. Insufficient data
    d. 1/2m

12. Indefinite integration of the function (e^{-x} cosx) with respect to x is, (constant of integration can be taken as 0)
    a. 1/2
    b. 1
    c. 2
    d. $2^{1/2}$

13. In a KdV equation $U_t+AUU_x+BU_{xxx} = 0$, the values of A and B are, respectively,
    a. 3/2 and 1/6
    b. 1/6 and 3/2
    c. Can take any value
    d. none

14. The equation u = w + ew$_x$ + Ae^2w^2 is
    a. Jukowsky transform
    b. Fourier transform
    c. Gardner transform
    d. none

15. The value of Heaviside unit step function H(t) for t<0, is
    a. 1
    b. 0
    c. Both (a) and (b)
    d. Oscillates between 0 and 1

16. Laplace transform of a function F is _____ Fourier transform of a function G.
    a. Equal to
    b. Less than
    c. Greater than
    d. None

17. Order of the p.d.e y(p2 + q2) = zq is
    a. 2
    b. 1
    c. 0
    d. None of these

18. Discrinant of a One dimensional wave equation r − t = 0 is,
    a. 2
    b. 3
    c. 4
    d. 1

19. For elliptic p.d.e, we get _____ families of complex characteristic curves.
    a. 2
    b. 3
    c. 4
    d. 1

20. The characteristic curves of (y2r − x2t = 0) are/is
    a. circle
    b. hyperbola
    c. Both (a) and (b)
    d. parabola

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[Answer question no.1 & any four (4) from the rest]

1. a. Solve the signal problem governed by the wave equation   6+4=10
        $u_{tt}=c^2 u_{xx}$, $u(x,0)=u_t(x,0)=0$, $u(0,t)=F(t)$, $t>0$: where $x>0$, $t>0$

    b. Find Fourier sine and cosine transform of x^{n-1}.

2. a. Derive KdV equation.   5+5=10
    b. Obtain a solution of wave equation in cylindrical coordinates
        by the method of separation of variables.

3. a. Mention the working rule for solving isoperimetric problems.   4+5+1
    b. Discuss the problem of Brachistochrone with solution.   =10
    c. Find Fourier sine transform of the fraction $1/x$.

4. a. Find the extremal for the functinal $(y^2 - y'^2 - 2y\sin x)$ which is   5+5=10
        itegrated over 0 to 90^0, with respect to x, and $y(0)=0$, $y(90^0)=1$.
    b. On what curve the functional $(-y^2 + y'^2 - 2yx)$ which is
        itegrated over 0 to 90^0, with respect to x, and $y(0)=0$, $y(90^0)=0$
        be extremized?

5. a. Transform spherical coordinates to cartesian coordinates.   5+3+2=10
    b. Write 3 forms of Euler's equation.
    c. Give two examples of partial differential equation which is
        elliptic and hyperbolic.

6. a. State and prove fundamental lemma of calculus of variations.   4+4+2
    b. State the conditions elaborately for which the a functional   =10
        attains maximum and minimum on any closed curve with
        some initial conditions.
    c. Define Linear functional with an example.

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7. a. If Fourier transform of $F_1(t)$ is equal to Laplace transform of $F_2(t)$, then how F_1 and F_2 are related to each other?   6+4=10
    b. Find the surface passing through the circle z=0, $x^2+y^2=1$ and satisfying the differential equation s=8xy.

8. a. Find the number of characteristic curves of the p.d.e $(x^2+2y)u_{xx}$ 3+3+4
        $+ (y^2-y+x)u_{yy} + x^2(y-1)u_{xy} - 3u_x + u = 0$, passing through the   =10
        point x=1, y=1.
    b. Determine the nature of the p.d.e $(u_{yy} - yu_{xx} + x^3u = 0)$.
    c. Determine the quadrants where the p.d.e $(yu_{xx} + xu_{yy} = 0)$ is hyperbolic.

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[4]