REV-01
MSP/01/05

2023/06

SET A

M.Sc. PHYSICS
SECOND SEMESTER
MATHEMATICAL PHYSICS
MSP – 201 [REPEAT]
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1x20=20***

1. The differential equation $x^2y''(x) + x\,y'(x) + (x^2 - n^2)\,y(x) = 0$ is called
   a. Legendre Equation
   b. Hermite Equation
   c. Bessel Equation
   d. Laguerre Equation

2. The value of kroneker delta symbol δ_i^i in 3-dimensional space is
   a. 1
   b. 3
   c. 6
   d. none of the above

3. The expression of $J_0(x)$ is
   a. $1 - \frac{1}{(1!)^2}\left(\frac{x}{2}\right)^2 + \frac{1}{(2!)^2}\left(\frac{x}{2}\right)^4 - \frac{1}{(3!)^2}\left(\frac{x}{2}\right)^6 + \cdots$
   b. $1 - \frac{1}{(2!)^2}\left(\frac{x}{2}\right)^2 + \frac{1}{(3!)^2}\left(\frac{x}{2}\right)^4 - \frac{1}{(4!)^2}\left(\frac{x}{2}\right)^6 + \cdots$
   c. $1 - \left(\frac{x}{2}\right)^2 + \frac{1}{2^2}\left(\frac{x}{2}\right)^4 - \frac{1}{3^2}\left(\frac{x}{2}\right)^6 + \cdots$..
   d. $1 - \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^4 - \left(\frac{x}{2}\right)^6 + \cdots$

4. The outer product of two mixed tensors $A_\gamma^{\alpha\beta}$ and B_α^ρ of rank 3 and 2 respectively followed by a contraction produces a new tensor of rank
   (a)0 (b) 1 (c) 3 (d) 5
   a. 0
   b. 1
   c. 3
   d. 5

5. The expression $J_{-n}(x)$ is
   a. $(-1)^{n-1}J_n(x)$
   b. $(-1)^n J_n(x)$
   c. $(-1)^{n+1}J_n(x)$
   d. $-J_n(x)$

6. The number of independent components of a symmetric tensor of rank 2 in n-dimensional space is
   a. n^2
   b. $\frac{n(n+1)}{2}$
   c. $\frac{n+1}{2}$
   d. $\frac{n(n-1)}{2}$

7. Which one of the following recurrence relation is true
   a. $2\,J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$
   b. $2\,J'_n(x) = -J_{n-1}(x) - J_{n+1}(x)$
   c. $2\,J'_n(x) = J_{n-1}(x) + J_{n+1}(x)$
   d. $2\,J'_n(x) = -J_{n-1}(x) + J_{n+1}(x)$

8. The value of $g_{\mu\nu}A^{\mu\lambda}$ is ($A^{\mu\lambda}$ is a contravariant tensor of rank 2 and $g_{\mu\nu}$ is the covariant metric tensor of rank 2)
   a. $A^{\nu\lambda}$
   b. $A_{\lambda\nu}$
   c. A_ν^λ
   d. A_ν^μ

9. $\left(\frac{1}{1-t}\right)e^{-\left(\frac{t}{1-t}\right)x}$ is the generating function of

a. Hermite polynomial
b. Laguerre polnomial
c. Bessel polynomial
d. Legendre polynomial

10. If $(G,\cdot)$ is a group such that $(ab)^{-1} = a^{-1}b^{-1}$ for $\forall a, b \in G$, then G is a/an

a. commutative subgroup
b. Abelian group
c. Non-Abelian group
d. none of these

11. The value of $L_1\left(\frac{1}{2}\right)$ will be

a. $-\frac{1}{2}$
b. 0
c. 1
d. $\frac{1}{2}$

12. This is an abelian group { - 3 n : n ε Z } under?

a. division
b. subtraction
c. addition
d. multiplication

13. If $f(x) = 2 + x = \sum_{n=0}^{1} c_n\, p_n\,(x)$then the value of c_0 is

a. 1
b. 2
c. 0
d. -1

14. If K is kernel of a group homomorphism f : G → H, then which statement is not true?

a. K is an abelian subgroup of G
b. K is a normal subgroup of G
c. K = {e} for some homomorphism
d. K = G for some homomorphism

15. The value of P_2^1 (sin θ) at $\theta = \frac{\pi}{4}$ will be

a. $\frac{1}{2}$
b. $\frac{3}{2}$
c. $\frac{3}{4}$
d. $\frac{1}{4}$

16. If b and c are elements in a group G and if $b^5 = c^3 = e$, where e is the identity of G, then the inverse of $b^2cb^4c^2$ must be

a. $cb^2c^2b^4$
b. $c^2b^4cb^2$
c. cbc^2b^3
d. $b^4c^2b^2c$

17. The value of $\int_{-\infty}^{\infty} e^{-x^2}\,[H_1]^2 dx$ will be

a. $\sqrt{\pi}$
b. $\sqrt{\frac{\pi}{2}}$
c. $\sqrt{\pi}/2$
d. $2\sqrt{\pi}$

18. For every group G, the identity mapping I_G defined by $I_G\colon G \rightarrow G$, $I_G(x) = x,\ \forall\, x \in G$ is a/an

a. homomorphism of G onto itself
b. isomorphism of G onto itself
c. one-one mapping
d. none of these

19. If $f(x) = 1 + 2x = \sum_{n=0}^{1} c_n H_n(x)$ then the value of c_1 is

a. 1 b. 2

c. ½ d. 0

20. A subset H of a group $(G,*)$ is group if

a. $a, b \in H \Rightarrow a * b \in H$

b. $a \in H \Rightarrow a^{-1} \in H$

c. $a, b \in H \Rightarrow a * b^{-1} \in H$

d. H contains the identity element

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(Descriptive)

Time : 2 hrs. 30 mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. a. Obtain a general of order n of the Legendre second-order differential equation. 8+2=10
   b. Define Legendre polynomial $p_n(x)$ in differential form

2. a. Define Christofell's symbols of first and second kind. Show that, $\Gamma^{\sigma}_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\lambda,\mu\nu}$. 4+4+2 =10
   b. Prove that the addition and subtraction of two mixed tensors of rank 2 each produces a new mixed tensor of rank 2.
   c. If $g_{\mu\nu} = 0$ for $\mu \neq \nu$ and μ, ν, σ are unequal indices, find the value of $\Gamma^{\nu}_{\mu\mu}$.

3. a. Using the Rodrigue's formula of Hermite polynomial obtain an expression of $H_3(x)$ 4+4+2 =10
   b. Prove that the Hermite polynomial satisfies the recurrence relation $2nH_{n-1}(x) = H_n'(x)$.
   c. Find an expression of $H_{2n}(x)$ at $x = 0$.

4. a. Show that the additive group $(R, +)$ of real numbers is isomorphic to the multiplicative group $(R^{+}, \times)$ of positive real numbers 4+3+3 =10

b. Check whether the set of integers Z with the binary operation " $*$ " defined as $a * b = a + b + 1$ for $a, b \in Z$ is a group or not.

c. Define kernel, range and null space of a linear transformation.

5. **a.** Define Volterra Integral Equation of the first kind. 2+6+2 =10

b. Reduce the boundary value problem to Fredholm Integral Equation $y''(x) + x\, y(x) = 1$, where $y(0) = 0,\ y(1) = 0$.

c. Express the above Fredholm Integral Equation into its 2nd kind

6. **a.** Explain the characteristics of an abelian group? 5+5=10

b. Let $G = \{1, -1, i, -i\}$, which forms a group under multiplication and Z is the group of all integers under addition. Prove that the mapping f onto G such that $f(x) = i^n\ \forall n \in Z$ is a homomorphism.

7. **a.** Express the polynomial $2\, p_2(x) + 3\, p_1(\, x)$ into the Laguerre polynomial. 4+4+2 =10

b. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{\pi x}{2}} \sin x$.

c. Prove that $L_n(0) = 1$.

8. Work out the symmetry group of a square. How many elements does it have? Construct the multiplication table. 2+2+6 =10

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P.T.O.