

Time: 30 min.

Objective

Marks: 20

Choose the correct answer from the following:

$$1 \times 20 = 20$$

- The special Theory of Relativity was given in which year?
 - 1906
 - 1905
 - 1907
 - 1908
 - The special relativity proves the existence of
 - Speed of light in vacuum is constant
 - Time is absolute
 - Ether medium
 - Speed of light is variable
 - A double pendulum consist of two point unequal masses m_1 and m_2 , respectively. The KE of the double pendulum is-
 - $\frac{1}{2} m_2 (\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2} m_1 (\dot{y}_1^2 + \dot{y}_2^2)$
 - $\frac{1}{2} m_1 (\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2} m_2 (\dot{y}_1^2 + \dot{y}_2^2)$
 - $\frac{1}{2} m_2 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_1 (\dot{x}_2^2 + \dot{y}_2^2)$
 - $\frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$
 - Hamiltonian H is in general
 - $H(q, \dot{q}, t)$
 - $H(p, \dot{q}, t)$
 - $H(q, \dot{p}, t)$
 - $H(q, p, t)$
 - The total number of variables in the Lagrangian $L(\dot{x}, t)$
 - $N+1$
 - $2N$
 - $N-1$
 - $2N+1$
 - For a monatomic molecules, the number of degrees of freedom-
 - 2
 - 3
 - 4
 - 5
 - Two point masses m_1 and m_2 are connected by a mass-less string hanging over a smooth pulley (Atwood Machine). The equation of motions-
 - $\ddot{x} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$
 - $\ddot{x} = \left(\frac{m_1}{m_1 + m_2} \right) g$
 - $\ddot{x} = \left(\frac{-m_2}{m_1 + m_2} \right) g$
 - $\ddot{x} = \left(\frac{2m_1 m_2}{m_1 + m_2} \right) g$
 - If the poisson bracket $\{x, p\} = 1$, then the poisson bracket $\{x^2, p\}$ is?
 - $2x$
 - $2p$
 - 1
 - 0

9. Hamilton's canonical equations of motion for a conservative system are
- $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$
 - $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$
 - $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$
 - $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$
10. A constant force F is applied on a relativistic scalar particle of mass m . If the particle is at rest at $t=0$, then the velocity at any instant will be
- $\frac{F t}{\sqrt{F^2 t^2 + m^2 c^2}}$
 - $\frac{F c t}{\sqrt{F^2 c^2 t^2 + m^2}}$
 - $\frac{F c t}{\sqrt{F^2 t^2 + m^2 c^2}}$
 - $\frac{F t}{\sqrt{F^2 + m^2 c^2}}$
11. The angular momentum in two body problem is
- $p_\theta = \mu r \dot{\theta}$
 - $p_\theta = \mu r \dot{\theta}^2$
 - $p_\theta = \mu r^2 \dot{\theta}$
 - $p_\theta = \mu r^2 / \dot{\theta}$
12. If a dynamical variable F is explicitly independent of time t , then the variable satisfies
- $\{H, F\} = \frac{dF}{dt}$
 - $\{H, F\} = -\frac{dF}{dt}$
 - $\{F, H\} = 1$
 - $\{F, H\} = 0$
13. A charge particle q is moving in an electromagnetic field with a velocity \vec{v} . The generalized potential U is-
- $q \Phi - \frac{q}{c} (\vec{A} \cdot \vec{v})$
 - $\frac{1}{2m} [q \Phi - \frac{q}{c} (\vec{A} \cdot \vec{v})]$
 - $\frac{1}{2m} q \Phi$
 - $q \Phi$
14. Two photon approach each other. Their relative velocity will be
- 0
 - c
 - $2c$
 - $-c$
15. The Lagrange's equations of motion are second degree, then the degree's of freedom will be?
- $2N+1$
 - N
 - $2N-1$
 - $N+1$
16. The generalized coordinate q_k of a classical system with Lagrangian L is said to be cyclic if
- $\frac{\partial L}{\partial q_k} = \dot{q}_k$
 - $\frac{\partial L}{\partial q_k} = 0$
 - $\frac{\partial L}{\partial q_k} = \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{q}_k} \right)$
 - $\frac{\partial L}{\partial \dot{q}_k} = 0$
17. The kinetic energy of a particle moving in free space is
- $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2)$
 - $T = \frac{1}{2} m (\dot{r}^2 + \dot{\theta}^2 + \dot{z}^2)$
 - $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2)$
 - $T = \frac{1}{2} m (r^2 \dot{\theta}^2 + \dot{z}^2)$
18. The rocket ship contracted to its length by 99%. Its velocity will be
- $0.1c$
 - $0.2c$
 - $0.3c$
 - $0.9c$

19. The Hamiltonian H is in general

- a. $H(q, p, t)$ b. $H(q, \dot{q}, t)$
c. $H(\dot{q}, p, t)$ d. $H(p, t)$

20. The value of the bracket $\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\}$

- a. 0 b. 1
c. -1 d. 2

-- -- --

[Descriptive]

Time : 2 hrs. 30 mins.

Marks : 50

[Answer question no. 1 & any four (4) from the rest]

1. A Lagrangian for a particular system

8+2=10

$$L = \frac{m}{2}(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2) - \frac{K}{2}(ax^2 + 2bxy + cy^2),$$

where a, b, c are arbitrary constants?

(a) What are the equations of motion for x-coordinate?

(b) Examine the cases $a = b = c$

2. Show that Poisson bracket $\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0$. 10

3. Write down the Lorentz transformation relations for inertial frame of reference. Using this transformation show that $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$ is invariant. 10

- 5+5=10
4. Show that the following transformations are canonical.
- (a) $P = \frac{1}{2}(p^2 + q^2)$, $Q = \tan^{-1}\left(\frac{q}{p}\right)$
- (b) $P = q \cot p$, $Q = \log\left(\frac{1}{q} \sin p\right)$
- 4+6=10
5. (a) What are the postulates of Special Relativity?
 (b) Using special theory of relativity, derive the relativistic energy relation $E^2 = p^2c^2 + M_0^2c^4$ where symbols have their usual meanings.
- 10
6. Construct the Lagrangian of a simple pendulum. Find the equations of motions and its time-period.
- 10
7. Using Hamilton-Jacobi equation derive Kepler's Law of orbit.
- 10
8. Two small blocks, each of mass M , attached to two identical springs of spring constant $k/2$. One of the springs is attached to a wall and other with block. The masses slide along the surface and the friction is negligible. Find the frequency of normal modes of the system.

$\equiv \equiv \star \star \equiv \equiv$

[4]