REV-01
MSM/48/16/19

2022/12

SET B

M.SC. MATHEMATICS
THIRD SEMESTER
CONTINUUM MECHANICS & HYDRODYNAMICS
MSM – 304

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. Full Marks : 70

(Objective)

Time: 30 min. Marks: 20

Choose the correct answer from the following: ***1X20=20***

1. Lagrangian's rotational tensor are formulates by
   a. $w = \frac{1}{2}(u\nabla_X + \nabla_X . u)$
   b. $w = (u\nabla_X - \nabla_X . u)$
   c. $w = \frac{1}{2}(u\nabla_X \mp . u)$
   d. $w = \frac{1}{2}(u\nabla_X - \nabla_X . u)$

2. In the cubic polynomial of σ the $\frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ij})$ is known as
   a. 1^{st} stress invariant
   b. 2^{nd} stress invariant
   c. 3^{rd} stress invariant
   d. Not exist

3. Eulerian's infinitesimal stain tensor are formulated by
   a. $e_{ij} = \frac{1}{2}(k - k_c)$
   b. $e_{ij} = \frac{1}{2}(k + k_c)$
   c. $e_{ij} = (k + k_c)$
   d. $e_{ij} = (k - k_c)$

4. A fluid motion is incompressible means
   a. $\nabla \times q \neq 0$
   b. $\Delta . q \neq 0$
   c. $\nabla \times q = 0$
   d. $\nabla . q = 0$

5. $\frac{\delta^2\varphi}{\delta x^2} + \frac{\delta^2\varphi}{\delta y^2} + \frac{\delta^2\varphi}{\delta z^2} = 0$ is the equation of continuity of
   a. Euler equation of continuity
   b. Hamiltonian equation of continuity
   c. Laplace equation of continuity
   d. None of these

6. A fluid motion is irrotational means
   a. $\nabla . q = 0$
   b. $\nabla . q \neq 0$
   c. $\nabla \times q \neq 0$
   d. $\nabla \times q = 0$

7. Poisson's ratio is
   a. $\mu = \frac{\beta}{\alpha}$, β is lateral strain and α is longitudinal stain
   b. $\mu = \frac{\alpha}{\beta}$, β is lateral strain and α is longitudinal stain
   c. $\mu = -\frac{\beta}{\alpha}$, β is lateral strain and α is longitudinal stain
   d. None of these

8. Strain Energy Formula is given by
   a. $v = -\frac{F\delta}{2}$
   b. $v = \frac{F\delta}{2}$
   c. $v = -\frac{Fx}{2}$
   d. $v = \frac{Fx}{2}$

9. Actual fluid is
a. Real fluid
b. Viscous Fluid
c. Compressible fluid
d. All of the above

10. E is known as
a. Constant of Elasticity
b. Young Modulus
c. Bulk viscosity
d. None of the above

11. $\frac{d}{dt} = \frac{\delta}{\delta t} + q.\nabla$ means
a. Equation of Laplace
b. Equation of continuity in Cartesian coordinate
c. Equation of continuityin incompressible fluid
d. None

12. Which one is the correct Cauchy's deformation tensor?
a. $G = F_C . F$
b. $C_{ij} = H_C . H$
c. $L_G = F_C . F$
d. None of the above

13. Chose the correct Lagrangian's finite stain tensor
a. $L_G = \frac{1}{2}(F_C.F - I)$
b. $L_G = \frac{1}{2}(F_C.F + I)$
c. $L_G = (F_C.F - I)$
d. None of these

14. Is the Green's finite strain tensor and Lagrangian finite strain tensor are
a. Same
b. Not same
c. Reverse
d. No realties between them

15. Choose the correct Eulerian's finite stain tensor
a. $E_A = \frac{1}{2}(I + H_C.H)$
b. $E_A = \frac{1}{2}(I + H_C.H)$
c. $E_A = \frac{1}{2}(I - H_C.H)$
d. All of the above

16. A material is that same property in all directions at a point is known as
a. homogeneity
b. Isotropy
c. Anisotropy
d. None of these

17. A material having identical properties at a point is known as
a. homogeneity
b. Isotropy
c. Anisotropy
d. None of these

18. In the cubic polynomial of σ the $|\sigma_{ij}| = \det \Sigma$ is a known as
a. 1st stress invariant
b. 2nd stress invariant
c. 3rd stress invariant
d. Not exist

19. $(\sigma_{12}, \sigma_{13}, \sigma_{21}, \sigma_{23}, \sigma_{31}, \sigma_{32})$ are knows as
a. Shearing Stress
b. Normal Stress
c. Both a and b
d. None

20. A material is called .... w.r.t those property which are directional at a point.
a. Homogeneity
b. Isotropy
c. Anisotropy
d. None of these

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. a. State the stress vector and stress tensor at a point? Establish the relationship between the stress vector and stress tensor? 5+5=10

   b. The stress tensor values at a point P are $\Sigma = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$, determine the traction on the plane at a point whose unit normal is $\hat{n} = \frac{2}{3}\hat{e_1} - \left(\frac{2}{3}\right)\hat{e_2} + \left(\frac{1}{3}\right)\hat{e_3}$. From the above problem determine the components perpendicular to the plane, also calculate the magnitude of traction.

2. a. How you will established force and moment equilibrium. Also define the stress tensor symmetry. 5+5=10

   b. The stress tensor at a point consider P is given by the axes $ox_1x_2x_3$ by the values $\sigma_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ Determine the principal stresses values and the principal stress directions represented by the axes $ox_1^*x_2^*x_3^*$

3. a. A displacement field is specified by $u = X^2{}_1X_2\hat{e_1} + (X_2 - X_3{}^2)\hat{e_2} + X^2{}_2X_3\hat{e_3}$. Determine the relative displacement vector du in the direction of the $-X_2$axis at $p\ (1, 2, -1)$. Determine the relative displacement $u_{Q_i} - u_p$ for $Q_1(1, 1, -1)$, $Q_2(1, \frac{3}{2}, -1)$, $Q_3(1, \frac{7}{4}, -1)$ and $Q_4(1, \frac{15}{8}, -1)$ and compare their directions with the direction of du 5+5=10

   b. A displacement field is given by $u = X_1X_3{}^2\hat{e_1} + X_1{}^2X_2\hat{e_2} + X_2{}^2X_3{}^2\hat{e_3}$, Determine independently the material deformation gradient F and the material displacement gradient J and verify $J = F - I$.

4. a. Define material displacement gradient and spatial displacement gradients. Discussed and give mathematical description the Green's deformation tensor and Cauchy's deformation tensor. 5+5=10

b. A continuum body undergo the deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$ where A is constant. Compute the deformation tensor G and use this to determine the Lagrangian finite strain tensor L_{ij}. Also calculate the material displace gradient J and use this tensor to determine the Lagrangian finite strain tensor L_G.

5. State and Proof Navier-Stokes Equation? 2+8=10

6. State and Proof Generalized Hook's Law? 3+7=10

7. a. Establish the relationship between the local time rate of change and individual time rate of change. 5+5=10

   b. Prove that liquid motion is possible when velocity at (x, y, z) is given by $u = \frac{3x^2-r^3}{r^5}$, $v = \frac{3xy}{r^5}$, $w = \frac{3xz}{r^5}$ where $r^2 = x^2 + y^2 + z^2$ and the sream lines are the intersection of the surfaces $(x^2 + y^2 + z^2)^2 = c(y^2 + z^2)$, by the plane passing through OX

8. a. Derive the Euler's dynamical equation of motion. 7+3=10

   b. Find the stream lines and path lines of the particles for the dimensional velocity field $u = \frac{x}{1+t}$, $v = y$, and $w = 0$

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