

M.Sc. MATHEMATICS  
THIRD SEMESTER  
GENERAL MATHEMATICS I  
MSM – 306 MDC  
[USE OMR SHEET FOR OBJECTIVE PART]

**SET  
B**

Duration : 3 hrs.

Full Marks : 70

( Objective )

Time: 30 min.

Marks: 20

Choose the correct answer from the following:

1X20=20

- The angle between the vectors  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  is
  - $\cos^{-1}\left(\frac{1}{3}\right)$
  - $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
  - $\cos^{-1}(\sqrt{3})$
  - None of these
- The unit vector in the direction of vector  $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$  is
  - $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$
  - $\frac{1}{\sqrt{14}}(2\hat{i} - 3\hat{j} + \hat{k})$
  - $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$
  - None of these
- Let A be a set contains n elements, then its power set P(A) contains:
  - n elements
  - $2^n$  elements
  - $n^2$  elements
  - None of these
- If  $f(x) = b \frac{x-a}{b-a} + a \frac{x-b}{a-b}$ , then
  - $f(a) + f(b) = f(a+b)$
  - $f(a) - f(b) = f(a+b)$
  - $f(a) + f(b) = f(a-b)$
  - None of these
- The remainder of the sum  $1! + 2! + 3! + \dots + 100!$  upon divided by 4 is
  - 0
  - 1
  - 2
  - None of these
- The remainder of  $111^{333} + 333^{111}$  upon divided by 7 is
  - 0
  - 1
  - 2
  - None of these
- $x \log x - x$  is the solution of
  - $\int \log x \, dx$
  - $\int x \log x \, dx$
  - $\int x^2 \log x \, dx$
  - None of these
- The value  $\int (\sin x + \cos x) \, dx$  is
  - $\cos x + c$
  - $\sin x + c$
  - $\tan x + c$
  - None of these

9. The value  $\int e^{\tan^{-1} x} \frac{1}{1+x^2} dx$  is
- $e^{\tan^{-1} x} + c$
  - $\frac{1}{1+x^2} + c$
  - $e^{\sin^{-1} x} + c$
  - None of these
10. Which of the following is/are uncountable
- $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$
  - $\{x \in \mathbb{N} : 0 \leq x \leq 1\}$
  - $\{x \in \mathbb{Q} : 0 \leq x \leq 1\}$
  - None of these
11. Let A be a non-empty set. The power set of A is a countable set if and only if
- A is finite.
  - A is countable.
  - Either A is finite or countable.
  - Data is insufficient.
12. Consider the following statements:  
P: The set  $\{x \in \mathbb{Q} : 1 < x < 2\}$  is uncountable.  
Q: The subset of uncountable set is uncountable
- P true, Q false
  - P false, Q true
  - P true as Q is true
  - P and Q both are false.
13. The value of  $\gcd(a, 0)$  is
- a
  - |a|
  - 0
  - 1
14. If  $\gcd(a, b) = d$ , the  $\gcd(ka, kb)$  is
- kd
  - $|k|d$
  - $k|d|$
  - None of these
15. Suppose A and B are two countably infinite set. The cardinality of  $A \times B$  is
- Finite
  - $\aleph_0$
  - c
  - Data is insufficient
16. If  $A_1, A_2, \dots, A_{n-1}$  are countable sets and  $A_n$  is uncountable then
- $\bigcup_{k=1}^{n-1} A_k$  is uncountable
  - $\bigcup_{k=1}^n A_k$  is countable
  - $\bigcup_{k=1}^{n-1} A_k$  is countable
  - None of these
17. Which of the following is/are true:
- Power set of any set is always countable
  - Union of countable sets is countable.
  - Intersection of countable set is countable.
  - None of these
18. Which of the following equation(s) has a solution:
- $6x - 9y = 5$
  - $9x + 21y = 12$
  - $34x - 98y = 67$
  - None of these
19. If  $f(x) = x + |x|$ , then  $f(-3)$  is equal to
- 6
  - 0
  - 6
  - None of these
20. The direction cosines of  $\hat{i} + \hat{j} - 2\hat{k}$
- (1, 1, -2)
  - (-1, -1, 2)
  - $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$
  - $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$



**( Descriptive )**

Time : 2 hrs. 30 mins.

Marks : 50

**[ Answer question no.1 & any four (4) from the rest ]**

1. Integrate

2×5=10

i.  $\int \frac{x^2+4}{x^2+2x+3} dx$

ii.  $\int (3x-2)\sqrt{x^2-x-1} dx$

2. a. Find the common solution of

6+4=10

$$\begin{aligned}x &\equiv 1 \pmod{7} \\x &\equiv 6 \pmod{10} \\x &\equiv 2 \pmod{11}\end{aligned}$$

b. Find the last two digits of  $3^{256}$ .

3. a. Find the unit vector in the direction of the sum of the vectors

4+3+3  
=10

$$\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k} \text{ and } \vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}.$$

b. Show that the points  $A(2\hat{i} - \hat{j} + \hat{k})$ ,  $B(\hat{i} - 3\hat{j} - 5\hat{k})$ ,  $C(3\hat{i} - 4\hat{j} - 4\hat{k})$  are the vertices of a right-angled triangle.

c. If  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ , then show that the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are perpendicular.

4. a. Which of the following sets of functions are uncountable? Explain.

5+5=10

- (i)  $\{f \mid f: \mathbb{N} \rightarrow \{1,2\}\}$
- (ii)  $\{f \mid f: \{1,2\} \rightarrow \mathbb{N}\}$
- (iii)  $\{f \mid f: \mathbb{N} \rightarrow \{1,2\}, f(1) \leq f(2)\}$
- (iv)  $\{f \mid f: \{1,2\} \rightarrow \mathbb{N}, f(1) \leq f(2)\}$
- (v)  $\mathbb{R} - \mathbb{N}$

b. Show that  $[0, 1]$  is uncountable.

5. a. Find the perpendicular distance of a point  $A(2, 3)$  from the line

2+4+4  
=10

$$3x - 4y + 1 = 0.$$

b. Find the angle between the line  $2x + y - 3 = 0$  and  $x + 3y + 2 = 0$ .

- c. Prove that the four points of intersection of the lines  $2x - y + 1 = 0$  &  $x - 2y + 3 = 0$  with the axes lie on a circle. Find its center and radius.

6. Evaluate

2×5=10

(i)  $\int e^{\sin^{-1} x} \frac{1}{\sqrt{1-x^2}} dx$

(ii)  $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$

(iii)  $\int \frac{1}{x\sqrt{x^4-1}} dx$

(iv)  $\int \sqrt{\frac{1+x}{1-x}} dx$

(v)  $\int x^3 e^x dx$

7. a. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are  $\hat{i} + 2\hat{j} - \hat{k}$  and  $-\hat{i} + \hat{j} + \hat{k}$  respectively, in the ratio 2:1 (i) Internally (ii) Externally.

4+3+3  
=10

- b. Find the unit vector perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , where  $\vec{a} = 3\hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

- c. Prove that - For any two vectors  $\vec{a}$  and  $\vec{b}$ ,  $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ .

8. a. Determine all solutions of  $256x + 116y = 2$ .

5+5=10

- b. Prove that - There are infinitely primes.

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