REV-01
MSP/38/19/23

2022/12

SET A

M.Sc. MATHEMATICS
THIRD SEMESTER
GENERAL MATHEMATICS I
MSM – 306 MDC

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. Which of the following is/are uncountable
   a. $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$ b. $\{x \in \mathbb{N} : 0 \leq x \leq 1\}$
   c. $\{x \in \mathbb{Q} : 0 \leq x \leq 1\}$ d. None of these
2. Let A be a non-empty set. The power set of A is a countable set if and only if
   a. A is finite. b. A is countable.
   c. Either A is finite or countable. d. Data is insufficient.
3. Consider the following statements:
   P: The set $\{x \in \mathbb{Q} : 1 < x < 2\}$ is uncountable.
   Q: The subset of uncountable set is uncountable
   a. P true, Q false b. P false, Q true
   c. P true as Q is true d. P and Q both are false.
4. The value of $\gcd(a, 0)$ is
   a. a b. $|a|$
   c. 0 d. 1
5. If $\gcd(a, b) = d$, the $\gcd(ka, kb)$ is
   a. kd b. $|k|d$
   c. $k|d|$ d. None of these
6. Suppose A and B are two countably infinite set. The cardinality of $A \times B$ is
   a. Finite b. $\aleph_0$
   c. c d. Data is insufficient
7. If $A_1, A_2, \cdots A_{n-1}$ are countable sets and A_n is uncountable then
   a. $\bigcup_{k=1}^{n-1} A_k$ is uncountable b. $\bigcup_{k=1}^{n} A_k$ is countable
   c. $\bigcup_{k=1}^{n-1} A_k$ is countable d. None of these
8. Which of the following is/are true:
   a. Power set of any set is always countable b. Union of countable sets is countable.
   c. Intersection of countable set is countable. d. None of these

9. Which of the following equation(s) has a solution:
   a. $6x - 9y = 5$
   b. $9x + 21y = 12$
   c. $34x - 98y = 67$
   d. None of these

10. If $f(x) = x + |x|$, then $f(-3)$ isequal to
   a. 6
   b. 0
   c. -6
   d. None of these

11. The unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is
   a. $\sqrt{14}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$
   b. $-2\hat{\imath} - 3\hat{\jmath} + \hat{k}$
   c. $\frac{1}{\sqrt{14}}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$
   d. None of these

12. The direction cosines of $\hat{\imath} + \hat{\jmath} - 2\hat{k}$
   a. $(1, 1, -2)$
   b. $(-1, -1, 2)$
   c. $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$
   d. $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

13. Let A be a set contains n elements, then its power set $P(A)$ contains:
   a. n elements
   b. 2^nelements
   c. n^2 elements
   d. None of these

14. If $f(x) = b\frac{x-a}{b-a} + a\frac{x-b}{a-b}$, then
   a. $f(a) + f(b) = f(a + b)$
   b. $f(a) - f(b) = f(a + b)$
   c. $f(a) + f(b) = f(a - b)$
   d. None of these

15. The angle between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ is
   a. $\cos^{-1}\left(\frac{1}{3}\right)$
   b. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   c. $\cos^{-1}(\sqrt{3})$
   d. None of these

16. The remainder of the sum $1! + 2! + 3! + \cdots + 100!$ upon divided by 4 is
   a. 0
   b. 1
   c. 2
   d. None of these

17. The remainder of $111^{333} + 333^{111}$ upon divided by 7 is
   a. 0
   b. 1
   c. 2
   d. None of these

18. The value $\int (\sin x + \cos x)\, dx$ is
   a. $\cos x$ +c
   b. $\sin x$ +c
   c. $\tan x$ +c
   d. None of these

19. The value $\int e^{\tan^{-1} x} \frac{1}{1+x^2}\, dx$ is
   a. $e^{\tan^{-1} x} + c$
   b. $\frac{1}{1+x^2}$ +c
   c. $e^{\sin^{-1} x} + c$
   d. None of these

20. $x \log x - x$ is the solution of
   a. $\int \log x\ dx$
   b. $\int x \log x\ dx$
   c. $\int x^2 \log x\ dx$
   d. None of these

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

2×5=10

1. Integrate

   i. $\int \frac{x^2+4}{x^2+2x+3} dx$

   ii. $\int (3x-2)\sqrt{x^2-x-1}\, dx$

6+4=10

2. a. Find the common solution of

$$x \equiv 1 (\text{mod } 7)$$
$$x \equiv 6 (\text{mod } 10)$$
$$x \equiv 2 (\text{mod } 11)$$

   b. Find the last two digits of 3^{256}.

4+3+3 =10

3. a. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.

   b. Show that the points $A(2\hat{\imath} - \hat{\jmath} + \hat{k}), B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k}), C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$ are the vertices of a right-angled triangle.

   c. If $\vec{a} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then show that the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

5+5=10

4. a. Which of the following sets of functions are uncountable? Explain.

   (i) $\{f \mid f: \mathbb{N} \to \{1,2\}\}$
   (ii) $\{f \mid f: \{1,2\} \to \mathbb{N}\}$
   (iii) $\{f \mid f: \mathbb{N} \to \{1,2\}, f(1) \leq f(2)\}$
   (iv) $\{f \mid f: \{1,2\} \to \mathbb{N}, f(1) \leq f(2)\}$
   (v) $\mathbb{R} - \mathbb{N}$

   b. Show that $[0, 1]$ is uncountable.

2+4+4 =10

5. a. Find the perpendicular distance of a point $A(2, 3)$ from the line $3x - 4y + 1 = 0$.

   b. Find the angle between the line $2x + y - 3 = 0$ and $x + 3y + 2 = 0$.

c. Prove that the four points of intersection of the lines $2x - y + 1 = 0$ & $x - 2y + 3 = 0$ with the axes lie on a circle. Find its center and radius.

6. Evaluate **2×5=10**

(i) $\int e^{a\sin^{-1}x} \frac{1}{\sqrt{1-x^2}}\, dx$

(ii) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}\, dx$

(iii) $\int \frac{1}{x\sqrt{x^4-1}}\, dx$

(iv) $\int \sqrt{\frac{1+x}{1-x}}\, dx$

(v) $\int x^3 e^x\, dx$

7. a. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively, in the ratio 2:1 (i) Internally (ii) Externally. **4+3+3 =10**

b. Find the unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = 3\hat{\imath} - 7\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

c. Prove that – For any two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.

8. a. Determine all solutions of $256x + 116y = 2$. **5+5=10**

b. Prove that – There are infinitely primes.

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