REV-01
MSP/41/21/26

2023/01

SET A

M.Sc. PHYSICS
FIRST SEMESTER
CLASSICAL MECHANICS
MSP – 103

[USE OMR SHEET FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(**Objective**)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1X10=10***

1. A constant force F is applied on a relativistic scalar of rest mass "m" particle. The particle is at rest at t=0, then the velocity at any instant will be
   a. $\frac{F\,t}{\sqrt{F^2\,t^2+m^2c^2}}$
   b. $\frac{F\,c\,t}{\sqrt{F^2t^2+m^2c^2}}$
   c. $\frac{F\,c\,t}{\sqrt{F^2\,c^2\,t^2+m^2c^2}}$
   d. $\frac{F\,t}{\sqrt{F^2\,c^2\,t^2+\,m^2c^2}}$

2. A particle moves in a plane under a conservative force field. Its kinetic energy will be
   a. $\frac{1}{2}m\,(\dot{r}^2+r\,\dot{\theta}^2)$
   b. $\frac{1}{2}m\,(\dot{r}^2+\dot{\theta}^2)$
   c. $\frac{1}{2}m\,(\dot{r}^2+r^2\,\dot{\theta})$
   d. $\frac{1}{2}m\,(\dot{r}^2+r^2\,\dot{\theta}^2)$

3. If the Poisson bracket $\{q, p\} = 1$, then the Poisson bracket $\{q + p,\ p^2\}$ is?
   a. $2\,p$
   b. p
   c. q
   d. 0

4. Hamilton's canonical equations of motion for a conservative system are
   a. $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$
   b. $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$
   c. $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$
   d. $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$

5. If a dynamical variable F is explicitly independent of time t, then the variable satisfies
   a. $\{H, F\} = \frac{dF}{dt}$
   b. $\{F, H\} = 1$
   c. $\{H, F\} = -\frac{dF}{dt}$
   d. $\{F, H\} = 0$

6. The kinetic energy of a particle is double its rest mass energy. Its moving mass will be
   a. Two times its rest mass
   b. Equal to its rest mass
   c. Three times its rest mass
   d. Four times its rest mass

7. The Poisson bracket of $\{p_i\ ,\ L_j\}$ is-
   a. $\epsilon_{ijk}p_k$
   b. $\epsilon_{ijk}L_k$
   c. L_k
   d. p_k

8. Two photons approach each other. Their relative velocity will be

a. 0
b. c
c. 2c
d. c/2

9. The Hamiltonian of a system with n degree of freedom is given by $H(q_1, \ldots, q_n; p_1, \ldots, p_n; t)$ with an explicit dependence on time. Which one of the following is correct?

a. Different phase trajectories cannot intersect each other.
b. H always represents the total energy of the system and is a constant of motion.
c. The equations $\dot{q}_i = \partial H/\partial p_i$, $\dot{p}_i = -\partial H/\partial q_i$ are not valid since H has explicit time dependence.
d. Any initial volume element in phase space remains unchanged in magnitude under time evolution.

10. A planet of mass m moves in the inverse square central force field of the Sun of mass M. If the semi-major axes of the orbit are "a" and "b", respectively, the total energy of the planet will be

a. $-\frac{G M}{(a+b)}$
b. $-\frac{G m}{(a+b)}$
c. $-\frac{G m M}{\sqrt{(a+b)}}$
d. $-\frac{G m M}{(a+b)}$

(Descriptive)

Time : 1 hr.15 mins. Marks: 25

[Answer question no.1 & any two (2) from the rest]

1. A particle moves in plane under a central potential V (r, θ, z). Find the Lagrange's equations of motion. **5**

2. a. A particle of mass m moves inside a bowl. If the surface is given by the equation $z = \frac{1}{2} a\,(x^2 + y^2)$, where a is a constant, find the Lagrange's equations of motion. **6+4=10**

   b. The Hamiltonian of a particle of unit mass moving in the xy-plane is given by $H = x\,p_x - y\,p_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$ in suitable units. Find the values of "a" for which the quantity (a $p_x - 3\,y$) is a constant of motion?

3. a. Show that if two events are simultaneous in one inertial frame then, these two events are not simultaneous in another inertial frame which is moving with a relativistic velocity relative to former. **10**

   b. Using velocity addition theorem prove that the speed of light in vacuum is the upper limit of speed of any object.

4. Show that Poisson bracket $\{A,\ \{B\ ,\ C\}\} + \{B\ ,\ \{C\ ,\ A\}\} + \{C\ ,\ \{A\ ,\ B\}\} = 0$. **10**

5. Prove that the following transformation relations are canonical. **10**

   (a) $P = \frac{1}{2}(p^2 + q^2),\ Q = \tan^{-1}\left(\frac{q}{p}\right)$

   (b) $P = q \cot p\ ,\ Q = \log\left(\frac{1}{q}\sin p\right)$

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