REV-01
MSP/34/11/14

2022/12

SET C

M.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PYSICS II
MSP – 301

[USE OMR FOR OBJECTIVE PART]

Duration : 3 hrs. **Full Marks : 70**

PART-A: Objective

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. By using the recurrence formula of Legendre polynomial, state that which of the following relation is correct?

a. $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+2)(2n+3)}$

b. $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$

c. $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+3)}$

d. $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$

2. The group of order<4 is:
   a. Always cyclic
   b. Always non-cyclic
   c. May or may not be cyclic
   d. Cyclic under some conditions

3. The recurrence formula for Bessel function of the form $\frac{d}{dx}[x^{-n}J_n(x)]$ is equal to ----------.
   a. $x^n J_n(x)$
   b. $-x^{-n}J_{n+1}(x)$
   c. $x^n J_{n+2}(x)$
   d. $x^n J_{n+2}(x)$

4. The generating function of Laguerre polynomial $L_n(x)$ is ------.
   a. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n(n-1)!}t^n$
   b. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n(n+1)!}t^n$
   c. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n!}t^n$
   d. $\sum_{n=0}^{\infty} \frac{L_n(x)}{(n-1)!}t^n$

5. What is the value of $\int_{-\infty}^{+\infty} f(x)\delta(x)dx$?
   a. $f(0)$
   b. $f(x)\delta(x)$
   c. $f(x)$
   d. None of these

6. The formula for Legendre polynomial in the form $P_n'(x) - xP_{n-1}'(x)$ is equal to ----------.

a. nP_{n-1} b. nP_{n+2}

c. nP_{n-2} d. nP_{n+1}

7. What is the Laplace Transform of f(t)δ(t-a)?

a. f(a) b. $f(a)e^{-as}$

c. $f(a)e^{as}$ d. None of these

8. Which of the following is correct?

a. δ(-x)=-δ(x) b. δ(ax)=(1/a)δ(x)

c. xδ(x)=1 d. f(x)δ(x-a)=f(a)δ(x-a)

9. What is the value of (x-a)δ(x-a)?

a. 1 b. 0

c. δ(x)/a d. aδ(x)

10. If every subgroup of a group is normal subgroup, then the group is:

a. Non-abelian b. Abelian

c. Necessarily cyclic d. None of the above

11. The following integro-differential equation is a ----------.

$$u'''(x) = Sinx - x + \int_0^{\pi/2} xtu'(t)dt;\ u(0) = 1, u'(0) = 0\ \&u''(0) = -1$$

a. Fredholm integro-differential equation & linear form

b. Fredholm integro-differential equation & non-linear form

c. Volterra integro-differential equation & linear form

d. Volterra integro-differential equation & non-linear form

12. If the Kernel $K'(x,t) = K(t.x)$, it is

a. asymmetric b. continuous

c. discontinuous d. symmetric

13. Set of integers (positive, negative and zero) forms

a. Semi group but not a group b. Group

c. Group which is a subgroup of the group of real numbers d. None of these

14. Which of the following is correct?

a. δ' (-x)=-δ' (x) b. δ' (-x)=δ' (x)

c. Both (a) and (b) d. None of these

15. The orthogonal properties of Laguerre polynomial L_n is given by

a. $\int_0^{\infty} e^x L_n(x)L_m(x)dx$ b. $\int_{-\infty}^{\infty} e^{-x} L_n(x)L_m(x)dx$

c. $\int_0^{\infty} e^{-x} L_n(x)L_m(x)dx$ d. $\int_{-\infty}^{\infty} e^x L_n(x)L_m(x)dx$

16. The value of Hermite polynomial $H_3(x)$ is
    a. $(8x^3 - 12x)$
    b. $(8x^2 - 12x^2)$
    c. $(8x^3 - 12x^3)$
    d. $(8x^3 - 12)$

17. The solution of $P_n(x)$and $Q_n(x)$is a series of one of the following kind.
    a. Both $P_n(x)$and $Q_n(x)$are non-terminating.
    b. Both $P_n(x)$and $Q_n(x)$are terminating.
    c. $P_n(x)$is non-terminating and $Q_n(x)$is terminating.
    d. $P_n(x)$is terminating and $Q_n(x)$non-terminating.

18. (G,*) is a group and a∈G; O(a) = 45. Find the order of a^{15}
    a. 10
    b. 15
    c. 9
    d. 5

19. If an electron has to face 3 double slits to arrive at the screen, in how many paths the electron can follow to reach the screen?
    a. 2
    b. 4
    c. 6
    d. 8

20. The equation $g(x) = f(x) + \int_a^b dt\ K(x,t) f(t)$is a
    a. Volterra equation of first kind
    b. Volterra equation of second kind
    c. Fredholm equation of first kind
    d. Fredholm equation of second kind

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PART-B : Descriptive

Time : 2 hrs. 30mins. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. **a.** Using the following second- order ordinary differential equation with the boundary conditions, convert the boundary value problem into a Fredholm integral equation. **6+4=10**

$$y''(x) + P(x)y'(x) + Q(x)y(x) = f(x)$$

$$x = a; y(a) = \alpha$$
$$y = b; y(b) = \beta$$

b. If $u(x) = x^2 + x^3$ ia a solution of the Fredholm integral equation, $u(x) = x^3 - x^2 - 2x + \alpha \int_{-1}^{+1} (xt^2x^2\, t)\, u(t)\, dt$, Find α.

2. **a.** In the following equations, classify each of them as Fredholm or Volterra or Singular; linear or non-linear; homogenous or non-homogenous. Justify the reason. **5+5=10**

i. $f(x) = \lambda \int_{\alpha(x)}^{\beta(x)} K(x,t)\, u(t)dt$

ii. $u(x) = e^x + \int_0^x t^2 \cos x\, u(t)dt$

b. Express the following function in Fourier- Legendre expansion.

$$f(x) = \begin{bmatrix} 0 \\ x^2 \end{bmatrix} \begin{matrix} -1 \le x \le 0 \\ 0 < x < 1 \end{matrix}$$

3. **a.** Prove that **3+3+4 =10**

(i) $$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

(ii) $$J_2'(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$$

b. Prove the orthogonality of Hermite polynomial.

4. **a.** Prove that **5+5=10**

$$\frac{1 - z^2}{(1 - 2xz + z^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1) P_n(x) z^n$$

b. Prove that

$$\int_{-1}^{+1} [P_n(x)]^2\, dx = \frac{2}{2n+1}$$

5. Let (G,°) be a group and c∈G. Define a binary composition * on G by a*b=a°b°c for a,b∈G. Show that (G,*) is a group with c-1 as the identity element. **10**

6. Define dirac delta function with figure and state six properties of dirac delta function. Find the Fourier Transform and Laplace Transform of dirac delta function. **5+5=10**
[Note: Do Not include Fourier Transform and Laplace Transform as a property].

7. Show that a family of block functions with Δ width, converges to dirac delta function for reduced Δ. Define dirac delta function as a limit of gaussian function. If Δ(x,a) is defined by Δ(x,a)=0,1/2a and 0 for x<-a,-a<x<a and x>0 respectively. Then show that $\delta(x) = \lim_{a\to 0} \Delta(x,a)$. **5+3+2 =10**

8. a. Explain Regular Representation of a group with example. Show that group of order 2 is always cyclic. **5+3+2 =10**

   b. Construct group tables for G={1,w,w2} and S={-1,1,i,-i} and state two properties of group table. Prove that inverse of the product of 2 elements of a group is the product of inverses in reverse order.

   c. Explain the significance of path integral technique.

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