

**M.Sc. MATHEMATICS
FOURTH SEMESTER
GRAPH THEORY
MSM-401**

Duration : 3 hrs.

Full Marks: 70

[PART-A: Objective]

Time : 20 min.

Marks : 20

Choose the correct answer from the following:

1X20=20

1. What is the number of edges present in complete graph K_n having n vertices.
 - a. $\frac{n(n+1)}{2}$
 - b. $\frac{n(n-1)}{2}$
 - c. n^2
 - d. None
2. Which of the following is true for any simple connected graph with more than 2 vertices
 - a. No two vertices have same degree
 - b. At least two vertices have same degree
 - c. At least 3 vertices have same degree
 - d. All vertices have same degree
3. A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is
 - a. A multiple of 4
 - b. Even
 - c. Odd
 - d. congruent to 0 mod 4, or 1 mod 4
4. A graph is said to be isomorphic if
 - a. The same numbers of vertices
 - b. The equal numbers of vertices have equal degrees
 - c. The equal numbers of edges
 - d. All of the above

5. What is the maximum number of edges in a bipartite graph having 12 vertices?
- a. 24
 - b. 38
 - c. 36
 - d. 32
6. A Graph contains m edges can be decomposed in
- a. $2^m - 1$ ways
 - b. $2^{m-1} - 1$ ways
 - c. 2^m ways
 - d. None
7. The number of Hamiltonian Circuits in a complete graph of n vertices
- a. $\frac{(n-1)!}{2}$
 - b. $\frac{n!}{2}$
 - c. $\frac{(n-2)!}{2}$
 - d. None of these.
8. Conditions of planarity for n numbers of vertices and e numbers of edges
- a. $e < 2n - 6$
 - b. $e < 2n + 6$
 - c. $e \leq 3n - 6$
 - d. None of these
9. Let n_1 and n_2 be the number of edges of graphs G_1 and G_2 respectively, then the number of edges of $G_1 \cup G_2$ if G_1 and G_2 are connected graphs
- a. $n_1 + n_2$
 - b. $n_1 + n_2 - 1$
 - c. n_1
 - d. n_2
10. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true?
- a. $v = e$
 - b. $v = e + 1$
 - c. $v + 1 = e$
 - d. None of these

11. Which of the following statements is/are TRUE for undirected graphs?
P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.
- P only
 - Q only
 - Both P and Q
 - Neither P nor Q
12. A cycle on n vertices is isomorphic to its Complement. The value of n is
- 3
 - 4
 - 5
 - 6
13. A connected undirected graph containing n vertices and $(n-1)$ edges
.....
- Cannot have cycles
 - Must contain at least one cycle
 - Can contain at most two cycles
 - Must contain at least two cycles
14. A graph is _____ if it has at least one pair of vertices without a path between them.
- Completed
 - Connected
 - Disconnected
 - Trivial
15. Total number of trees with 5 vertices
- 2
 - 3
 - 4
 - 5
16. What is the radius of the Petersen graph
- 2
 - 3
 - 4
 - All of these
17. Let $\kappa(G)$ = vertex connectivity, $\lambda(G)$ = edge connectivity and $\delta(G)$ = minimum degree of graph G . Then
- $\kappa(G) \leq \delta(G) \leq \lambda(G)$
 - $\lambda(G) \leq \delta(G) \leq \kappa(G)$
 - $\delta(G) \leq \kappa(G) \leq \lambda(G)$
 - $\kappa(G) \leq \delta(G) < \lambda(G)$

18. Let G be a simple graph with every pair of vertices is connected. Then G is

- a. Trivial
- b. Complete
- c. Disconnected
- d. Self-complementary

19. Let $G = C_n$. Then

- a. There is path of length n
- b. There is a closed path of length n
- c. G is bipartite
- d. None of the above

20. A graph is called a _____ if it is a connected acyclic graph

- a. Cyclic graph
- b. Regular graph
- c. Tree
- d. Trivial graph

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(PART-B : Descriptive)

Time: 2 HRS 40 MINS

Marks : 50

[Answer question no.(1) & any four (4) from the rest]

1. a. Write a brief History of Graph Theory. 5+5=10
b. Prove that Simple Graph with n -vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges

2. a. With examples of each define union, intersection, Edge deletion, vertex deletion, fusion of a Graph. 5+5=10
b. Prove that the number of labeled trees with n vertices ($n \geq 2$) is n^{n-2} .

3. a. Show that any connected Graph with n -vertices and $(n-1)$ edges are a tree. 5+5=10
b. Show that the spanning tree in a connected Graph of n vertices and e edges has $(n-1)$ tree branches and $(e-n+1)$ chords

4. 5+5=10
a. Show that The maximum vertex connectivity one can achieve with a graph G of n vertices and e edges, $e \geq n-1$ is the integral part of the number $\frac{2e}{n}$ i.e. $\left\lfloor \frac{2e}{n} \right\rfloor$
b. Show that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-set

5. a. Prove that the complete Graph of five vertices is non-planar. 5+5=10
b. Define complete Graph. Show that in a complete Graph with n vertices there are $\frac{n-1}{2}$ edges disjoint Hamiltonian circuit if n is odd no, $n \geq 3$.

6. a. Prove that -Every tree with two or more vertices is two chromatic. 5+5=10
b. Let a and b two non-adjacent vertices in a Graph G . Let G' be a

graph obtained by adding an edge between a and b . Let G'' be a simple Graph obtained from G by fusing the vertices a and b together and replacing sets of parallel edges with single edges then prove that $P_n(\lambda)$ of $G = P_n(\lambda)$ of G' + $P_{n-1}(\lambda)$ of G''

7. a. If G is a simple Graph with largest vertex Δ then prove that $\Delta \leq x'(G) \leq \Delta + 1$. 5+5=10
 b. Define Trivial Graph, Pseudo graph, Trivial Graph, pendent vertex, isolated vertex. Also define and explain characteristics of isomorphism of graph with a suitable diagram
8. a. Prove that every connected Graph has at most one spanning Tree. 5+5=10
 b. Prove that a connected Graph G is an Euler Graph if all vertices of G are even degree.

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