REV-01 2022/07
MSM/45/50

# M.Sc. MATHEMATICS
## FOURTH SEMESTER
## ADVANCED PARTIAL DIFFERENTIAL EQUATION
## MSM-402

**Duration : 3 hrs.** **Full Marks: 70**

## ( PART-A: Objective )

**Time : 20 min.** **Marks : 20**

*Choose the correct answer from the following:* *1X20=20*

1. A general partial differential equation of second order for a function of two independent variable is hyperbolic if
   a. $S^2 - 4RT < 0$
   b. $S^2 - 4RT > 0$
   c. $S^2 - 4RT = 0$
   d. None of the above

2. The partial differential equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$ is
   a. Hyperbolic type
   b. Ellptic type
   c. Parabolic type
   d. None of the above

3. The number of eigen value of $U_{xx} + U_{yy} = U_{zz}$ is
   a. Three
   b. Two
   c. one
   d. zero

4. The characteristics equation of $y^2 r - x^2 t = 0$ is
   a. $x^2 - y^2 = c_1, x^2 - y^2 = c_2$
   b. $x^2 + y^2 = c_1, x^2 - y^2 = c_2$
   c. $x^3 + y^3 = c_1, x^2 - y^2 = c_2$
   d. $x^3 - y^3 = c_1, x^2 - y^2 = c_2$

5. The Linear Hyperbolic equation in canonical form is
   a. $Rr + Ss + Tt + f(x, y, z, p, q) = 0$
   b. $L(z) = f(xy)$
   c. $L(z) = f(x, y)$
   d. None of the above

6. In Partial Differential Equation the $s = ?$
   a. $s = \frac{\partial u}{\partial x}$
   b. $s = \frac{\partial u}{\partial y}$
   c. $s = \frac{\partial u}{\partial t}$
   d. $s = \frac{\partial^2 u}{\partial x \partial y}$

7. The wave equation in three dimension is
   a. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$
   b. $\frac{\partial^3 u}{\partial x^2} + \frac{\partial^3 u}{\partial y^2} + \frac{\partial^3 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$
   c. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c}\frac{\partial^2 u}{\partial t^2}$
   d. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 u}{\partial t^2}$

8. In Fourier Sine series the constant $b_n$ is

a. $b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx, n = 1,2,3.......$

b. $b_n = \frac{2}{l}\int_0^l f(x)\sin nx dx, n = 1,2,3.......$

c. $b_n = \frac{-2}{l}\int_0^l f(x)\sin nx dx, n = 1,2,3.......$

d. $b_n = \frac{-2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx, n = 1,2,3.....$

9. The Solution of Heat equation $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to the boundary condition $u_x(0,t) = u_x(a,t) = 0\forall t$ ,initial condition is $u(x,o) = f(x)\forall x$ is

a. $u(x,t) = \sum_{n=1}^{a} E_n \cos\frac{n\pi x}{a} e^{-c_n^2 t}$

b. $u(x,t) = \frac{E_0}{2} + \sum_{n=1}^{a} E_n \cos\frac{n\pi x}{a} e^{-c_n^2 t}$

c. $u(x,t) = -\sum_{n=1}^{a} E_n \cos\frac{n\pi x}{a} e^{-c_n^2 t}$

d. $u(x,t) = \frac{E_0}{2} - \sum_{n=1}^{a} E_n \cos\frac{n\pi x}{a} e^{-c_n^2 t}$

10. A linear PDE of 2nd order in 3-independent variables $x_1, x_2, x_3$ is parabolic if

a. All eigen values are non zero

b. $|A| \neq 0$

c. $|A| = 0$

d. None of the above

11. For $0 < x < l$, then $f(x) = \frac{a_0}{2} + \sum_{n=1}^{a} a_n \cos\frac{n\pi x}{l}$ ,where $a_0 = \frac{2}{l}\int_0^l f(x)dx$,

$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx, n = 1,2,3.....$ This is known as

a. Fourier cosine series

b. Laplace equation

c. Fourier Sine series

d. None of the above

12. The solution of $\frac{\partial^2 z}{\partial x \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = f(x,y)$ which satisfies the boundary conditions that $z$ and $\frac{\partial z}{\partial x}$ are prescribed along curve C in the xy-plane is

a. $[z]_P = [wz]_Q - \int_Q^P wz(a\,dy - b\,dx) + \int_Q^P \left(z\frac{\partial w}{\partial y}dy + w\frac{\partial z}{\partial x}dx\right) + \iint wf\,dx\,dy$

b. $[z]_P = [wz]_Q + \int_Q^P wz(a\,dy - b\,dx) + \int_Q^P \left(z\frac{\partial w}{\partial y}dy + w\frac{\partial z}{\partial x}dx\right) + \iint wf\,dx\,dy$

c. $[z]_P = [wz]_Q - \int_Q^P wz(a\,dy - b\,dx) - \int_Q^P \left(z\frac{\partial w}{\partial y}dy + w\frac{\partial z}{\partial x}dx\right) + \iint wf\,dx\,dy$

d. None of the above

13. Subsidiary equations for the equation $r + 4t = 8xy$ are

a. $dpdy + 4dpdx - 8xydqdx = 0$ ; $dy + 4dx = 0$

b. $dpdy + 4dqdx - 8xydxdy = 0$ ; $(dy)^2 + 4(dx)^2 = 0$

c. $dqdx + 4dpdy - 8xydxdy = 0$ ; $dy + 4(dx)^2 = 0$

d. None of these

14. Subsidiary equations for the equation $yr + (x - y)s - xt = p - q$ are

a. $ydpdx - xdqdy - (p - q)dxdy = 0$ ; $y(dy)^2 - (x - y)dxdy - xdx = 0$

b. $ydpdy - xdpdx - (q - p)dxdy = 0$ ; $dy - (x - y)dxdy - xdx = 0$

c. $ydpdy - xdqdx - (p - q)dxdy = 0$ ; $x(dy)^2 - (x - y)dxdy - x(dx)^2 = 0$

d. None of these

15. The $\lambda$ quadratic for $5r + 6s + 3t + 2(rt - s^2) + 3 = 0$ is

a. $9\lambda^2 - 12\lambda + 4 = 0$

b. $9\lambda^2 - 12\lambda - 4 = 0$

c. $9\lambda^2 + 12\lambda - 4 = 0$

d. $9\lambda^2 + 12\lambda + 4 = 0$

16. The Euler's equation for the variational problem "Minimise $I[y(x)] = \int_0^1 (2x - xy - y')y' \, dx$ is

a. $2y'' - y = 2$

b. $2y'' + y = 2$

c. $y'' + 2y = 0$

d. $2y'' - y = 0$

17. Extremals $y = y(x)$ for the variational problem $I[y(x)] = \int_0^1 (y + y')^2 \, dx$ satisfies the differential equation

a. $y'' + y = 0$ b. $y'' - y = 0$

c. $y'' + y' = 0$ d. $y' + y = 0$

18. $\lambda$ quadratic for the equation $2s + (rt - s^2) = 1$ is

a. $\lambda^2 - 2\lambda + 1 = 0$ b. $\lambda^2 - 2\lambda - 1 = 0$

c. $\lambda^2 + 2\lambda - 1 = 0$ d. $\lambda^2 + 2\lambda + 1 = 0$

19. The functional $I[y(x)] = \int_0^2 (y'^2 + 6xy + x^3) dx$, $y(0) = 0, y(2) = 2$ can be extremised on the curve

a. $y = x$ b. $2y = x^3$

c. $y = x^2 - 6x$ d. $2y = x^3 - 2x$

20. The curve of shortest distance between two fixed points in a plane is

a. A straight line b. A circle

c. A cycloid d. None of these

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## ( PART-B : Descriptive )

Time: 2 HRS 40 MINS Marks : 50

[ Answer question no.(1) & any four (4) from the rest ]

1. (a)Write four difference between one dimensional Wave equation and Heat Equation? what is the Laplace equation in polar coordinate system? 4+1=5

(b)What are Monge's subsidiary equations for solving a partial differential equation of the form $Rr+Ss+Tt=V$ where $R,S,T,V$ are functions of $x,y,z,p,q$ ?Also find Monge's subsidiary equations for the partial differential equation $(r-s)y+(s-t)x+q-p=0$ 2+3=5

2. Reduce $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=0$ to canonical form. 10

3. What is Riemann Method of solution of general linerar hyperbolic equation of second order?Find the solution valid when $x,y>0, xy>1$ of the equation $\frac{\partial^2 z}{\partial x \partial y}=\frac{1}{x+y}$ such that $z=0, p=\frac{2y}{x+y}$ on the hyperbola? 1+9=10

4. What is Heat equation in two dimension in certasian coordinate system?Find the General solution of heat flow equation $k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ by the method of separation of variable? 2+8=10

5. (a)Find the curve on which the functional $\int_0^1\left[y'^2+12xy\right]dx$ with $y(0)=0$ and $y(1)=1$ can be extremised? 5+5=10

(b)What is a "Geodesic"on a surface? Show that the Geodesics on a plane are straight lines.

6. Find the characteristic equations of $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$ .Prove that 2+8=10

$\frac{\partial^2 z}{\partial y^2} = 0$. Also, reduces the above equation to canonical form.

7. For the partial differential equation $3s + rt - s^2 = 2$ find the roots of the $\lambda$ quadratic equation and the intermediate integrals associated with these roots. Hence find the complete solution of the equation? 2+3+5=10

8. Solve the Boundery value problem by the method of separation of variable 5+5=10

(a) $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, if $u(0, y) = 8e^{-3y}$

(b) $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, if $u(0, y) = 8e^{-3y} + 4e^{-5y}$

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