REV-01
MSM/58/63

2022/07

M.Sc. MATHEMATICS
SECOND SEMESTER
MECHANICS & TENSOR
MSM-204

Duration : 3 hrs. **Full Marks: 70**

(PART-A: Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: *1X20=20*

1. The radial unit vector in cylindrical coordinate system is
   a. $\hat{\rho} = \cos\phi\hat{i} - \sin\phi\hat{j}$ b. $\hat{\rho} = \cos\phi\hat{i} + \sin\phi\hat{j}$
   c. $\hat{\rho} = \sin\phi\hat{i} - \cos\phi\hat{j}$ d. $\hat{\rho} = \sin\phi\hat{i} + \cos\phi\hat{j}$

2. Position vector in spherical coordinate system is
   a. $\vec{r} = r\sin\theta\cos\varphi\hat{i} + r\sin\theta\sin\varphi\hat{j} - r\cos\theta\hat{k}$ b. $\vec{r} = r\sin\theta\cos\varphi\hat{i} - r\sin\theta\sin\varphi\hat{j} + r\cos\theta\hat{k}$
   c. $\vec{r} = r\sin\theta\cos\varphi\hat{i} + r\sin\theta\sin\varphi\hat{j} + r\cos\theta\hat{k}$ d. $\vec{r} = r\sin\theta\cos\varphi\hat{i} - r\sin\theta\sin\varphi\hat{j} - r\cos\theta\hat{k}$

3. The velocity expression in spherical coordinate system is
   a. $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\varphi}\hat{\varphi}\sin\theta$ b. $\vec{v} = \dot{r}\hat{r} - r\dot{\theta}\hat{\theta} + r\dot{\varphi}\hat{\varphi}\sin\theta$
   c. $\vec{v} = \dot{r}\hat{r} - r\dot{\theta}\hat{\theta} - r\dot{\varphi}\hat{\varphi}\sin\theta$ d. $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} - r\dot{\varphi}\hat{\varphi}\sin\theta$

4. The equation $\frac{d}{dt}\left\{\frac{\partial(T-V)}{\partial q_0}\right\} - \frac{\partial(T-V)}{\partial q_0} = 0$, where T, V are kinetic and potential energy is known as
   a. Lagrangian equation b. Generalized principle
   c. Equation of compound pendulum d. None of the above

5. Generalized coordinates is a
   a. Special no of coordinates for specifying the position of object or any other rigid body
   b. Minimum no of coordinates for specifying the position of object or any other rigid body
   c. Maximum no of coordinates for specifying the position of object or any other rigid body
   d. None of the above

6. The reversed effective forces acting on each particle of the body and the external forces of the system are in equilibrium.This statement is known as
   a. General statement of equation of motion
   b. General statement of compound pendulum
   c. D'Alembert's principle
   d. None of the above

7. The statement of Principle of virtual work is
   a. Virtual work is the work which is done on the system only by the constraint force
   b. Virtual work is the work which is done on the system only by the applied force

c. Virtual work is the work which is done on the system only by the equillibrium force

d. None of the above

8. One of the equation of motion of rigid body is

a. $\sum m\frac{d^2x}{dt^2} = \sum X$

b. $\sum m\frac{d^2x}{dt^2} = \sum Y$

c. $\sum m\frac{d^2x}{dt^2} = \sum Z$

d. $\sum m\frac{d^2x}{dt^2} = \sum (zX - xZ)$

9. The mathematical form of Impulsive force is

a. $I = \vec{F}dt$

b. $I = \vec{F}t$

c. $I = \int \vec{F}dt$

d. None of the above

10. Moment of Inertia of a Hollow sphere is

a. $\frac{2Ma^2}{3}$

b. $\frac{Ma^2}{4}$

c. $\frac{Ma^2}{3}$

d. $M\frac{4a^2}{3}$

11. The product of inertia with respect to the axes y and z is

a. $\sum myx$

b. $\sum myz$

c. $\sum mxz$

d. $\sum my^2z$

12. If PQ be a small portion of a thin uniform rod of mass M and length $2a.$ Also given $AP = x, PQ = \delta x$ .The mass of the small portio PQ is

a. δxM

b. $\frac{\delta x}{2a}$

c. $\frac{\delta x}{4a}M$

d. $\frac{\delta x}{2a}M$

13. In the expression $b_{ij}x^i{}_{,}{}^i$ is

a. Dummy suffix

b. Real Suffix

c. Summation convention

d. None of the above

14. Value of $A^i\delta^i{}_j = ?$

a. A^i

b. A^i

c. $A^i{}_j$

d. None of the above

15. Which of the following is correct

a. $\delta^p{}_q A^{qi}{}_m = A^{pi}$

b. $\delta^p{}_q A^{qi}{}_m = A^i{}_m$

c. $\delta^p{}_q A^{qi}{}_m = A^{pi}{}_m$

d. None of the above

16. Which of the following is correct

a. $[ij,k]=[ki,j]$

b. $[ij,k]=-[ji,k]$

c. $[ij,k]=[ji,k]$

d. None of the above

17. The Christoffel's Symbol of second kind are defined by

a. $\begin{Bmatrix} k \\ ij \end{Bmatrix} = g_{kh}[ij,h]$

b. $\begin{Bmatrix} k \\ ij \end{Bmatrix} = g^{kh}[ij,h]$

c. $\begin{Bmatrix} k \\ ij \end{Bmatrix} = g_{kh}[ih,k]$

d. None of the above

18. The metric in Euclidean three dimensional space is given by $ds^2 = dx^2 + dy^2 + dz^2$.

We know that $g_{11} = g_{22} = g_{33} = 1$ .In Cylindrical coordinate system

$g_{11} = ?, g_{22} = ?, g_{33} = ?$

a. $g_{11} = 1, g_{22} = r^2, g_{33} = 1$

b. $g_{11} = 1, g_{22} = -r^2, g_{33} = 1$

c. $g_{11} = 1, g_{22} = r^2, g_{33} = -1$

d. $g_{11} = -1, g_{22} = r^2, g_{33} = -1$

19. Kronecker delta is a tensor of rank

a. one

b. three

c. zero

d. two

20. The law of transformation for the tensor $B^{i}{}_{jk}$ is

a. $\overline{B}^{r}{}_{lq} = \dfrac{\partial \overline{x}^{l}}{\partial x^{i}} \dfrac{\partial x^{j}}{\partial \overline{x}^{r}} \dfrac{\partial x^{k}}{\partial \overline{x}^{q}} B^{i}{}_{jk}$

b. $\overline{B}^{r}{}_{lq} = \dfrac{\partial \overline{x}^{r}}{\partial x^{j}} \dfrac{\partial x^{i}}{\partial \overline{x}^{l}} \dfrac{\partial x^{k}}{\partial \overline{x}^{q}} B^{i}{}_{jk}$

c. $\overline{B}^{r}{}_{lq} = \dfrac{\partial \overline{x}^{r}}{\partial x^{i}} \dfrac{\partial x^{j}}{\partial \overline{x}^{l}} \dfrac{\partial x^{k}}{\partial \overline{x}^{q}} B^{i}{}_{jk}$

d. $\overline{B}^{r}{}_{lq} = \dfrac{\partial \overline{x}^{r}}{\partial x^{j}} \dfrac{\partial x^{i}}{\partial \overline{x}^{l}} \dfrac{\partial x^{k}}{\partial \overline{x}^{q}} B^{i}{}_{jk}$

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(PART-B : Descriptive)

Time: 2 HRS 40 MINS **Marks : 50**

[Answer question no.(1) & any four (4) from the rest]

1. Write the definition of Outer product and Inner Product of two tensor? What is the condition of Addition and substraction of two tensor? Prove that the Inner product of tensors $A^{p}{}_{q}$ and $B^{ij}{}_{l}$ is a tensor of rank three. 2+1+7=10

2. Find the Lagrange's equation of motion from D'Alemberts Principle? 10

3. What is three dimensional motion? Find the velocity and acceleration in Cylindrical coordinate system? 2+8=10

4. **Find the moment of Inertia for**
   (a)Thin Uniform Rod
   (b)Rectangular Lamina

5. Write the definition of Virtual work, virtual displacement, compound pendulum, moment of inertia and product of Inertia 2×5=10

6. What is Riemannian Geometry? Prove that Fundamental metric tensor is a covariant,symmetric, and a tensor of order two. 1+3+3+3=10

7. What do you think about the motion about a fixed axis?A uniform rod of mass m and length $2a$,can turn freely about one end which is fixed .It is started with angular velocity w from the position in which it hangs vertically. find the motion 2+8=10

8. What is Christoffel Symbol of 1st kind and 2nd Kind? Prove that Christoffel symbols are not tensor quantities. 4+6=10

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