Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number_____

Course_____ Semester_____

Paper Code _____Paper Title _____

Type of Exam: _____ (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

M.Sc. PHYSICS
THIRD SEMESTER
MATHEMATICAL PHYSICS-II
MSP-301

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Which one of the following is not the essential property to form the Group?
a. Closure Property **b.** Existence of identity element
c. Existence of inverse elements **d.** Commutative law

2. In path integral formalism, the fluctuation q(t) at the initial and final time (i.e. q(ti) and q(tf)) is:
a. > 0 **b.** < 0
c. 0 **d.** Infinite

3. Consider a set of two real numbers, 1 and -1 along with the simple binary operation multiplication will satisfy the property of:
a. Simple Group **b.** Abelian group
c. Non-abelian group **d.** None

4. Classically and quantum mechanically how many paths are possible in between initial(xi , ti) and final (xf ,tf) states respectively in path-integral formalism?
a. Only one in both cases **b.** Infinitely many and only one
c. Infinitely in both cases **d.** Only one and infinitely many

5. Consider the following options related to the symmetry operation of square,
1. $\sigma_u\ C_4 = m_x$
2. $C^3_4\ C_4 = E$
3. $m_x\ C^3_4 = \sigma_u$
4. $C^2_4\ C_4 = C^3_4$
Which will be the correct options? (symbols have their usual meaning)
a. Only 1 **b.** 1 and 2
c. 1,2 and 3 **d.** 1,2,3 and 4

6. If AB = C, then which of the following condition represents the representation of the Group? (where A, B, and C are the elements of Group and T be the collection of non-singular square matrices)
a. T(A) T(B) = T(C) **b.** T(A) T(C) = T(B)
c. T(B) T(C) = T(A) **d.** T(A) T(B) = - T(C)

7. Which of the following represents the correct relation relating to the initial(|i>) and final(|f>) state?
a. $|f\rangle = Exp\{(-i/\hbar)(t_i-t_f)H\}|i\rangle$ **b.** $|f\rangle = Exp\{(i/\hbar)(t_f-t_i)H\}|i\rangle$
c. $|f\rangle = Exp\{(-2i/\hbar)(t_f-t_i)H\}|i\rangle$ **d.** $|f\rangle = Exp\{(-i/\hbar)(t_f-t_i)H\}|i\rangle$

8. Which one of the following statement(s) is(are) related to field F,
1. F forms a Group under addition with 1 as additive identity.
2. F forms an abelian Group under addition with 1 as additive identity.
3. F forms an abelian Group under multiplication with 0 as a multiplicative identity.
4. Set of non zero elements of F forms an abelian group under addition with 1 as additive identity.

a. Only 1 **b.** 2 and 3
c. Only 4 **d.** None

9. Which of the following statement is correct related to Vector space V defined over field F,
1. Scalar multiplication by a with vector element A, i.e. aA belongs to V.
2. Scalar multiplication by a with vector element A, i.e. aA belongs to F.
3. Scalar elements and vector elements belong to F and V respectively.
4. a (A+B) = aA+aB belongs to V.

a. Only 1 **b.** Only 3
c. 1, 2 and 3 **d.** 1, 3 and 4

10. The following vectors are linearly ……,
U = (1,2,3), V = (2,0,-1), W = (1,-1,1) and X = (2,1,0)

a. Dependent **b.** Independent
c. Both a and b **d.** None

11. The solution of $P_n(x)$ and $Q_n(x)$ is a series of one of the following kind:

a. Both $P_n(x)$ and $Q_n(x)$ are terminating **b.** $P_n(x)$ is non-terminating and $Q_n(x)$ terminating
c. Both $P_n(x)$ and $Q_n(x)$ are non-terminating **d.** $P_n(x)$ is terminating and $Q_n(x)$ non-terminating

12. The value of Hermite polynomial $H_2(x)$ is:

a. 0 **b.** 1
c. $(4x^2 - 2)$ **d.** $(8x^3 - 12x)$

13. The recurrence formula for Bessel function of the form is equal to:

a. $x^n J_n{}'$ **b.** $x J_n{}'$
c. $x^n J'_{n-1}$ **d.** $x^n J_{n-1}$

14. In Volterra integral equation, at least one limit of the range of integration is……..

a. Constant **b.** Variable
c. Unity **d.** Undefined

15. Which of the following functions is oscillatory in nature and with a decreasing amplitude and varying time period?

a. Hermite function **b.** Bessel's function
c. Legendre's polynomial **d.** Both Bessel's function and Legendre's polynomial

16. The orthogonality condition of Hermite polynomial for (m s given by:

a. $\int_{-\infty}^{+\infty} H_m(x) H_n(x)\, e^{-x^2}\, dx = 0$ **b.** $\int_{-\infty}^{+\infty} \varphi_m(x)\, \varphi_n(x) dx = 0$

c. $\int_{-\infty}^{+\infty} \varphi_m(x)\, \varphi_n(x) e^{-x^2}\, dx = 0$ **d.** $\int_{-\infty}^{+\infty} H_m(x) H_n(x)\, e^{x^2}\, dx = 0$

17. The recurrence formula for Bessel function of the form is equal to:

a. $x^n J_n$ **b.** $x^n J_{n+1}$

c. $x^n J_{n+2}$ **d.** $x^n J_{n-1}$

18. The generating function of Laguerre polynomial is:

a. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n(n-1)!} t^n$ **b.** $\sum_{n=0}^{\infty} \frac{L_n(x)}{n\,(n+1)!} t^n$

c. $\sum_{n=0}^{\infty} \frac{L_n(x)}{n!} t^n$ **d.** $\sum_{n=0}^{\infty} \frac{L_n(x)}{(n-1)!} t^n$

19. By using the recurrence formula of Legendre polynomial, state that which of the following relation is correct?

a. $\int_{-1}^{+1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ **b.** $\int_{-1}^{+1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$

c. $\int_{-1}^{+1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n}{(2n-1)(2n+3)}$ **d.** $\int_{-1}^{+1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$

20. Choose the correct option from the following:

a. $\frac{d}{dx}(x^{-n}.J_n) = -x^{-n} J_{n+1}$ **b.** $\frac{d}{dx}(x^{n}.J_n) = -x^{-n} J_{n+1}$

c. $\frac{d}{dx}(x^{-n}.J_n) = x^{-n} J_{n+1}$ **d.** $\frac{d}{dx}(x^{-n}.J_n) = -x^{-n} J_{n-1}$

-- --- --

PART-B : Descriptive

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. What is symmetry transformation? Explain ALL the equivalent symmetry transformation of square. Draw the multiplication table and show that set of ALL the symmetry operations forms a group. (The symmetry operations are E, C_4, C_4^2, C_4^3, m_x, m_y, σ_μ , σ_v) 10

2. **a)** Define group properties with suitable examples. Write down the definition of Lorentz group. 5+5=10
b) Define vector space, inner product space and linear transformation with suitable examples.

3. Explain the path-integral formalism. Derive the expression of matrix element $\langle r_i \ldots . | r_f \rangle$ related to the path-integral representation. 10

4. Derive the path-integral for harmonic oscillator. 10

5. **a)** Prove that: 3+3+4=10
(i)

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$$

(ii)

$$J_2'(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$$

b) Prove the orthogonality of Bessel function.

6. **a)** Classify the following integral equation as Fredholm or Volterra integral equation, Linear or Non-linear and Homogenous or Non-homogeneous. Justify your answer. 4+3+3=10
(i)

$$u''(x) = \frac{1}{2}x^2 - \int_0^x (x-t)u^3(t)dt, u(0) = 1; u'(0) = 0$$

(ii)

$$u(x) = x + \int_{-\infty}^{0} Cos(x+t)u(t)dt$$

(iii)

$u(x) = x^2 + x^3$ is a solution of of the Fredholm integral

$$u(x) = x^2 - x^3 - 2x + \alpha \int_{-1}^{+1} xt^2 + x^2 t\, u(t)dt$$

equation

Find α.

7. **a)** Prove that 3+4+3=10

$$\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)P_n(x)z^n$$

b) Express the following function in Fourier-Legendre expansion.

$$f(x) = \begin{bmatrix} 0 & -1 \le x \le 0 \\ x^2 & 0 \le x \le 1 \end{bmatrix}$$

c) Prove that $J_n(x)$ is a coefficient of z^n in the expansion of $e^{x/2(z-\frac{1}{z})}$.

8. **a)** Prove that 4+6=10

$$\int_{-1}^{+1}[P_n(x)]dx = \frac{2}{2n+1}$$

b) Obtain the integral form of Linear harmonic oscillator equation by transformation of its differential form into homogenous Fredholm Integral equation of second kind.

= = *** = =