M.Sc. PHYSICS
First Semester
MATHEMATICAL PHYSICS-I
(MPH - 101)

Duration: 3Hrs. **Full Marks: 70**

Part-A (Objective) =20
Part-B (Descriptive) =50

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins. **Marks: 50**

Answer any *four* from *Question no.* 2 to 8
***Question no.* 1 is compulsory.**

1. (a) Solve Laplace's equation in spherical polar coordinates.

   (b) Express Laplace's equation in cylindrical coordinate.

   (8+2=10)

2. State and prove the theorem of diagonalization of a matrix. Find the eigen values, eigen vectors, model matrix and then diagonalize the following matrix. (5+5=10)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

3. State the necessary conditions for the vectors $X_1,\ X_1,\ X_1 \ldots . X_n$ to be dependent. Are the vectors $X_1 = (1,0,0)$, $X_2 = (0,1,0)$ and $X_3 = (0,0,1)$ are linearly dependent? Find the relation between the vectors $X_1 = (1,2,4)$, $X_2 = (2,-1,3)$, $X_3 = (0,1,2)$, $X_4 = (-3,7,2)$. (2+3+5=10)

4. State the Fourier theorem for any periodic, continuous function and write the Dirichlet conditions for Fourier series expansion. Find the Fourier series of $f(x)$ where (1+4+5=10)

$$f(x) = \begin{cases} 0 \ for - \pi < x < 0 \\ \frac{\pi}{4} x \ for - \pi < x < 0 \end{cases}$$

5. (a) Using Cauchy's Residue theorem find integral of $\oint \frac{4-3z}{z(z-1)(z-2)} dz$, where c is the circle $|z|=\frac{3}{2}$.

(b) Find the Taylor or Laurent series for $f(z)=\frac{1}{(z+1)(z+3)}$, when

(i) $0<|z+1|<2$ (ii) $|z|<1$

(4+6=10)

6. (a) State and prove Cauchy integral theorem.

(b) Use Cauchy's integral formula to evaluate $\oint \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}$ dz, where c is the circle $|z|=3$.

(4+6=10)

7. (a) Write the components of metric tensor in spherical polar coordinate. Show that the covariant metric tensor $g_{\mu\nu}$ is a symmetric tensor of rank 2.

(b) Define Dirac- δ function in one dimension. Show that,

$$\int_{-\alpha}^{+\alpha} \delta'(x) f(x) dx = -f'(0)$$

(5+5=10)

8. (a) Using tensor formalism prove that $\vec{a}.(\vec{b} \times \vec{c}) = \vec{b}.(\vec{c} \times \vec{a})$.

(b) Define Christofell's symbols of second kind. Show that

$$\Gamma^{\sigma}_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\lambda,\mu\nu}.$$

(c) What is a periodic Strum-Liouville system? Explain with an example.

(2+3+5=10)

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Duration: 20 minutes **Marks – 20**

(PART A - Objective Type)

I. Choose the correct answer: **1×20=20**

1. If $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$, the A^{-1} will be

   a) $\begin{bmatrix} i & 1 \\ 0 & i \end{bmatrix}$ b) $\begin{bmatrix} i & 0 \\ 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ d) $\begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$

2. The eigen values of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are

   a) $1, 0$ b) $1, 1$ c) $1, 2$ d) $0, 2$

3. Eigen values of a triangular matrix are the .... elements of the matrix.
   a) diagonal b) conjugate of diagonal
   c) off diagonal d) all of these

4. Two eigen vectors, A and B are orthogonal if
   a) $A^T B = 0$, b) $B^T A = 0$,
   c) both (a) and (b) d) none of these

5. The necessary and sufficient condition for matrix A to be Hermitian is
   a) $A = \bar{A}$, b) $A = \bar{A}^T$, c) $A = A^T$, d) $A = A^{-1}$

6. Laplace transform of (e^{at}) is equal to
   a) $\frac{1}{s}$ b) $\frac{1}{s-a}$ c) $\frac{a}{s^2+a^2}$ d) $\frac{s}{(s-a)^2}$

7. Fourier transform of integral, $F\left[\int_0^t f(t)\, dt\right]$ is equals to

   a) $i\omega F(\omega)$ b) $\frac{1}{i\omega} F(\omega)$ c) $\frac{i}{\omega} F(i\omega)$ d) $\frac{1}{a} F\left(\frac{\omega}{a}\right)$

8. If A_{ij} is an anti-symmetric tensor, then the value of the identity $\varepsilon_{ijk} A_{jk}$ will be
   a) 1 b) 0 c) -1 d) 3

9. If $g_{\mu\nu} A^\mu B^\nu = 0$, then the tensors A^μ and B^ν are said to be
   a) Alternate tensors b) Conjugate tensor
   c) Orthogonal tensor d) Symmetric tensor

10. If $g_{\mu\nu} = 0$ for $\mu \neq \nu$ and μ, ν, σ are unequal indices then, the value of $\Gamma^{\mu}_{\nu\sigma}$,

a) $\frac{1}{2}\frac{g_{\mu\mu}}{\partial x^{\nu}}$ b) 0 c) $-\frac{1}{2g_{\nu\nu}}\frac{g_{\mu\mu}}{\partial x^{\nu}}$ d) $\frac{1}{2}\frac{g_{\mu\mu}}{\partial x^{\mu}}$

11. The value of the identity $\delta_{ik}\varepsilon_{ikm}$ is

a) 3 b) +1 c) -1 d) 0

12. The inner product of two tensors $A^{\mu\nu}_{\sigma}$ and B^{λ}_{ρ} of rank 3 and 2 respectively, produces a tensor of rank

a) 5 b) 3 c) 2 d) 0

13. The wave equation is

a) $\nabla^2\phi = 0$ b) $\nabla^2\phi = \rho$ c) $\Box^2\phi = 0$ d) $\nabla^2\phi = k^2\phi$

14. Which of the following statement is incorrect for the Green's function G(x,t)?

a) G(x,t) is a continuous function of x.
b) The first derivative of Green's function is a discontinuous function.
c) Green's function is discontinuous at $x = t$.
d) Green's function is a characteristic of the given boundary conditions.

15. The value of integral $\int IzIdz$, where c is the straight line from z=-i to z=i is

a) 1 b) i c) 0 d) 2i

16. Let f (z)=$\frac{1}{(z-2)^4(z+3)^6}$ then z=2 and z=-3 are the pole of order is

a) 6 and 4 b) 2 and 3 c) 3 and 4 d) 4 and 6

17. The residue of $\frac{1+e^z}{\sin z + z\cos z}$ at z= 0 is

a) 1 b) -1 c) 0 d) 2

18. The integral $\oint \frac{e^{-z}}{z+1}$dz, where c is the circle IZI=$\frac{1}{2}$ is

a) 1 b) $\frac{1}{2}$ c) -1 d) 0

19. If f(z) = $\frac{\sin(z-a)}{(z-a)^4}$,then f(z) has a pole at z=a of order

a) 3 b) 4 c) 2 d) 5

20. The Cauchy-Riemann equation for f (z) =u(x, y) +iv(x, y) to be analytic are

a) $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=0, \frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$ b) $\frac{\partial U}{\partial x} = -\frac{\partial V}{\partial y}, \frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$

c) $\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}, \frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$ d) $\frac{\partial U}{\partial x} = -\frac{\partial V}{\partial y}, \frac{\partial U}{\partial y} = \frac{\partial V}{\partial x}$

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