8. (a) Let S^* be an extension of S. Prove that if S^* is consistent, then so is S. 4+6=10

(b) Describe $\mathcal{G}$ the extension of K_{L_G} with usual meaning in A. G. Hamilton's Logic for Mathematics.

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M.Sc. MATHEMATICS
FOURTH SEMESTER
MATHEMATICAL LOGIC
MSM-403 B

(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs. **Full Marks : 70**

[PART-A : Objective]

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1x20=20***

1. Find out the "True" statement variable.
   a. Square root of 5 is 2
   b. $3^0=1$
   c. Cabbage is a fruit
   d. Wood is a metal

2. Find out the statement form having truth value "F" for truth values "T" of p, q.
   a. $p \wedge q$
   b. $p \vee q$
   c. $p \rightarrow q$
   d. $p \wedge \sim q$

3. If the truth values of p and q are T and F respectively, then the truth value of $p \rightarrow q$ is:
   a. T
   b. F
   c. Cannot be said
   d. None of these

4. An extension of L is consistent if:
   a. $\vdash_L A$ and $\vdash_L (\sim A)$ for any wf A.
   b. Any wf A is not a theorem of L.
   c. For no wf A of L are both A and $(\sim A)$ theorems of the extension.
   d. A and $(\sim A)$ are theorems of the extension.

5. By which of the following connectives the truth values T and F can result in truth value T?
   a. $\leftrightarrow$
   b. $\wedge$
   c. $\rightarrow$
   d. $\vee$

6. For no natural number n is $n!$ equal to:
   a. 0
   b. 1
   c. 2
   d. 3

7. An argument form is a sequence which is:
   a. Finite
   b. Infinite
   c. Both
   d. None of these

8. Every atomic formula of $\mathcal{L}$ is a ________ of $\mathcal{L}$.
   a. wf
   b. Not a wf
   c. May or may not be a wf
   d. None of these

9. For any wfs A, B of L, $v(A \rightarrow B) = F$ if and only if:
   a. $v(A) = T$ and $v(B) = F$
   b. $v(A) = T$ and $v(B) = T$
   c. $v(A) = F$ and $v(B) = T$
   d. $v(A) = F$ and $v(B) = F$

10. If $\Gamma \vdash_L (A \rightarrow B)$ then pick up the right one.

a. A, B *are wfs of L*
b. Γ *is a set of wfs of L*
c. $\Gamma \cup \{A\} \vdash_L B$
d. $\Gamma \cup \{B\} \vdash_L A$

11. How many truth functions will be obtained for a statement form involving n different statement variables?

a. 2^n
b. 2^{2n}
c. 2^{n-1}
d. None of these

12. If $P(x) \equiv \{(-x)^2 = x^2\}$, where the domain is all integers, then the truth value is:

a. T
b. F
c. Cannot be said
d. None of these

13. ZF is a formal system of

a. Set theory
b. Group theory
c. Both (i) and (ii)
d. None of these

14. Peano's postulates are a _________ for the system of natural numbers.

a. MP
b. Set of Axioms
c. Extersions
d. None of these

15. A *wf*. $\mathcal{A}$ of $\mathcal{L}$ is closed if no variables occurs________ in $\mathcal{A}$.

a. Bound
b. Open
c. Free
d. None of these

16. P: Some people are intelligent. The symbol of the statement is:

a. $(\sim x)(M(x) \wedge S(x))$
b. $(\exists x)(M(x) \rightarrow S(x))$
c. $(\exists x)(M(x) \wedge S(x))$
d. None of these

17. P: Not all birds can fly. The symbol of the statement is:

a. $(\forall x)(B(x) \rightarrow F(x))$
b. $\sim(\exists x)(B(x) \rightarrow F(x))$
c. $\sim(\forall x)(B(x) \rightarrow F(x))$
d. None of these

18. Consider the following statements:

(i) A *wf*. of $\mathcal{A}$ of $\mathcal{L}$ is logically valid if $\mathcal{A}$ is true in every interpretation of $\mathcal{L}$.
(ii) $\mathcal{A}$ is contradictory if it is false in every interpretation.

a. (i) is true
b. (ii) is true
c. Both (i) and (ii) are true
d. Neither (i) nor (ii) are true

19. A *wf* A of L is a tautology if:

a. $v(A) = v(\sim A)$
b. $v(A) = F$
c. $v(A) = T$
d. A *is valid*

20. If $\Gamma \cup \{A\} \vdash_L B$ then which one is correct?

a. $\Gamma \cup \vdash_L \{A \rightarrow B\}$
b. $\vdash (A \rightarrow B)$
c. $\Gamma \vdash (\sim A \rightarrow A)$
d. None of the these

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(PART-B : Descriptive)

Time : 2 hrs. 40 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. (a) Show that $(\sim(p \wedge q))$ is logically equivalent to $((\sim p) \vee (\sim q))$. 4+6=10

(b) If $\mathcal{A}$ *and* $(\mathcal{A} \rightarrow \mathcal{B})$ are tautologies, then show that $\mathcal{B}$ is a tautology.

Show that the statement form $(((\sim p) \rightarrow q) \rightarrow (p \rightarrow (\sim q)))$ is a contradiction.

2. (a) Show that the pairs $\{\sim, \wedge\}, \{\sim, \vee\}$ *and* $\{\sim, \rightarrow\}$ are adequate sets of connectives. 5+5=10

(b) Investigate the validity of the argument form

$(p \rightarrow q), ((\sim q) \rightarrow r), r; \therefore p$

3. State a set of axiom schemes and a rule of inference for the formal theory L of propositional calculus. For any two *wfs* $\mathcal{A}$ *and* $\mathcal{B}$ of L prove that$(\sim\mathcal{B}$ $(\mathcal{B} \rightarrow \mathcal{A})$ is a theorem of L. 4+6=10

4. Define a valuation of L. State and prove the Soundness theorem for L. 2+8=10

5. State and prove the converse of the deduction theorem. 5+5=10

For any *wfs* A *and* B of L prove that $((\sim\mathcal{A} \rightarrow (\mathcal{A}) \rightarrow \mathcal{A})$ is a theorem of L.

6. (a) Define the following: 4+4+2=10

(i) Free term in a *wf* $\mathcal{A}$.
(ii) Interpretation I of $\mathcal{L}$.

(b) Prove that if in a particular interpretation I, the *wfs* $\mathcal{A}$ and $\mathcal{A} \rightarrow \mathcal{B}$ are true, then $\mathcal{B}$ is true.

(c) State the conditions where a *wf* of $\mathcal{L}$ is logically valid and contradictory.

7. (a) Translate the following into symbols. 5+3+2=10

(i) Not all birds can fly.
(ii) Anyone can do it.
(iii) Some people are intelligent.
(iv) There is an integer which is greater than every other integer.
(v) Not every function has derivative.

(b) Define the following:

(i) First order language
(ii) Atomic formula in $\mathcal{L}$.
(iii) A well formed formula in $\mathcal{L}$.

(c) Write the first order language of the following:

$x_1 + x_2 = x_1 x_2$, by using following symbols a_1 stands for identity, A_1^2 stands for =, f_1^1 stands for the function which takes each element to its inverse, f_1^2 stands for group operation.