M. Sc. MATHEMATICS
FIRST SEMESTER
ABSTRACT ALGEBRA - I
MSM – 103

Duration: 3 Hrs. **Marks: 70**

Part : A (Objective) = 20
Part : B (Descriptive) = 50

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. State and prove the fundamental theorem of group homomorphism. 3+7=10

2. a. Define subgroup of a group G. Union of two subgroups may not be a subgroup. Justify with an example. 4+6=10

b. State and prove the Lagrange's theorem.

3. Answer the following: 3+2+5 =10

a. Define ideal of a ring with example.

b. Define prime ideal of a ring.

c. Prove that an ideal P of a commutative ring R is prime iff R/P is an integral domain.

4. a. Define external direct product of groups. 3+7=10

b. Let G_1 and G_2 be two cyclic groups of order 2 and 3 respectively. Is $G_1 \times G_2$ cyclic? Justify with an example.

5. a. State the Eisenstein's criteria for a polynomial over a ring. 2+3+5 =10

b. Prove that the polynomial $f(x) = x^3 + x^2 - 2x - 1$ is irreducible over Q.

c. If R[x] is the ring of polynomials over a ring R, then prove that R is commutative iff R[x] is commutative.

6. a. Define solvable group. Show that S_3 is a solvable group. 4+6=10

b. Prove that a subgroup H of a solvable group is solvable.

7. a. Define Sylow p-subgroup. 2+2+6 =10

b. State Sylow's third theorem.

c. If o(G)=200, then find the Sylow p- subgroups of G.

8. a. Define cyclic group. 2+3+5 =10

b. Prove that cyclic group is abelian.

c. Prove that a subgroup of a cyclic group is cyclic.

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[PART-A : Objective]

Choose the correct answer from the following : 1×20=20

1. An isomorphism from a group G to itself is called
   a. Monomorphism
   b. Epimorphism
   c. Endomorphism
   d. Automorphism

2. If G is a cyclic group of order 6, then which of the following can be order of its subgroup
   a. 3
   b. 4
   c. 7
   d. 9

3. Let H be a subgroup of G, then which of the following is false
   a. $Ha = H \Leftrightarrow a \in H$
   b. $Ha = Hb \Leftrightarrow ab^{-1} \in H$
   c. $Ha = Hb \Leftrightarrow ab \in H$
   d. Ha is a subgroup of G iff $a \in H$

4. A group of order 15 is
   a. Abelian
   b. Non abelian
   c. Cannot be determined
   d. None of these

5. The set of zero divisors of $(Z_6,+,.)$ is
   a. {0}
   b. {0,2}
   c. {0,2,3}
   d. None of these

6. If G is a finite group of order n and a∈G is any element of order m. Then G is cyclic if
   a. $m > n$
   b. $m = n$
   c. $m < n$
   d. None of these

7. Which of the following is true
   a. UFD ⊆PID⊆ED
   b. PID⊆UFD⊆ED
   c. ED⊆ PID⊆UFD
   d. PID⊆ED⊆UFD

8. The additive inverse of (1, 2) in $Z_3 \times Z_5$ is
   a. (2, 1)
   b. (2, 3)
   c. (1, 3)
   d. None of these

9. Consider the polynomials $f(x) = 8x^3 + 6x + 1 \in Z[x]$ and $g(x) = 8x^3 + 6x + 2 \in Z[x]$, which of the these is primitive
   a. Only $f(x)$
   b. Only $g(x)$
   c. Both $f(x)$ and $g(x)$
   d. None of these

10. Any integral domain can be imbedded into a/an
    a. Integral domain
    b. Field
    c. Ring without unity
    d. None of these

11. If G is a group of order 10, then which of the following can be a class equation of G
    a. 1+1+2+2+2+2=10
    b. 1+1+1+2+3+3=10
    c. 1+1+1+2+5=10
    d. None of these

12. A homomorphism $f : G \to G'$ is one-one iff
    a. $Kerf \neq \{e\}$
    b. $Kerf = \{e\}$
    c. $Kerf = G$
    d. None of these

13. In a commutative ring R with unity, if all the non zero elements of R have multiplicative inverse, then it is called
    a. Field
    b. Skew field
    c. Integral domain
    d. Ideal

14. Let R be a commutative ring with unity. Then an ideal M of R is maximal iff R/M is a
   a. Skew field
   b. Field
   c. Ideal
   d. None of these

15. A finite group G is a p- group if and only if
   a. $o(G) = p^n$
   b. $o(G) = p^{n+1}$
   c. $o(G) = p^{n-1}$
   d. None of these

16. Which of the following is true.
   a. If G is a finite group and p is any prime such that p^k divides o(G) but p^{k+1} doesnot divide o(G), then there exists no subgroup of order p^k
   b. Any group of order 55 is abelian
   c. If H and K are two p- Sylow subgroups of a finite group G, then there exists an element $x \in G$ such that $H = xKx^{-1}$
   d. None of these

17. If G is a finite group with its centre as Z(G) and C(a) is the centralizer of any a∈G, then the class equation of G is defined as
   a. $o(G) = o[Z(G)] + \sum_{a \in Z(G)} \frac{o(G)}{o[C(a)]}$
   b. $o(G) = o[Z(G)] + \sum_{a \notin Z(G)} \frac{o(G)}{o[C(a)]}$
   c. $o(G) = o[Z(G)] + \sum_{a \in G} \frac{o(G)}{o[C(a)]}$
   d. None of these

18. Let H be a normal subgroup of G. Then G is solvable if
   a. H is solvable
   b. G/H is solvable
   c. Both H and G/H are solvable
   d. None of these

19. The number of subgroups of Z_{30} is
   a. 3
   b. 8
   c. 10
   d. 15

20. Z_n is a field if n is
   a. Prime
   b. An odd integer
   c. An even integer
   d. Any positive integer

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : ....

Semester : .... Roll No : ....

Enrollment No : .... Course code : ....

Course Title : ....

Session : 2017-18 Date : ....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*